Question1

The area of the triangle formed by the co-ordinate axes and a tangent to the curve $xy = a^2$ at the point (x_1, y_1) is ________ sq.units (where a, x_1 and y_1 are non zero) MHT CET 2025 (5 May Shift 2)

Options:

A. $\frac{a^2 x_1}{y_1}$

B. $\frac{a^2 y_1}{x_1}$

C. $2a^2$

D. $4a^2$

Answer: C

Solution:

- The equation of the tangent to the hyperbola $xy = a^2$ at (x_1, y_1) is:

$$x_1 y + y_1 x = 2a^2$$

- The intercepts on the axes are:
  - On x-axis: set $y = 0 \Rightarrow y_1 x = 2a^2 \Rightarrow x = \frac{2a^2}{y_1}$
  - On y-axis: set $x = 0 \Rightarrow x_1 y = 2a^2 \Rightarrow y = \frac{2a^2}{x_1}$
- The triangle's area with intercepts A and B is:

$$\text{Area} = \frac{1}{2} \times \left(\frac{2a^2}{y_1}\right) \times \left(\frac{2a^2}{x_1}\right) = \frac{1}{2} \times \frac{4a^4}{x_1 y_1}$$

- But on $xy = a^2$, $x_1 y_1 = a^2$. So:

$$\text{Area} = \frac{1}{2} \times \frac{4a^4}{a^2} = 2a^2$$

Question2

The minimum value of the slope of the tangent to curve $y = x^3 - 3x^2 + 2x + 93$ is MHT CET 2025 (5 May Shift 2)

Options:

A. 1

B. -1

C. 2

D. -2

Answer: B

Solution:

The question asks for the minimum value of the slope of the tangent to the curve $y = x^3 - 3x^2 + 2x + 93$.

The slope of the tangent at any x is given by the derivative of y:

$$\frac{dy}{dx} = 3x^2 - 6x + 2$$

To find the minimum value, set the derivative of this slope to zero:

$$\frac{d}{dx}(3x^2 - 6x + 2) = 6x - 6 = 0 \implies x = 1$$

Substitute $x = 1$ into the slope formula:

$$3(1)^2 - 6(1) + 2 = 3 - 6 + 2 = -1$$

Question3

The approximate value of $\frac{1}{(2{\cdot}002)^2}$ is MHT CET 2025 (5 May Shift 2)

Options:

A. 0.2495

B. 0.2595

C. 0.2095

D. 0.2392

Answer: A

Solution:

The question asks for the approximate value of $\frac{1}{(2.002)^2}$.

Calculating this:

$$(2.002)^2 \approx 4.008004$$

$$\frac{1}{4.008004} \approx 0.2495$$

Question4

A spherical balloon is filled with 4500π cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage has begun, is MHT CET 2025 (5 May Shift 2)

Options:

A. $\frac{9}{7}$

B. $-\frac{2}{9}$

C. $\frac{9}{2}$

D. $\frac{2}{9}$

Answer: D

Solution:

Answer: $\frac{2}{9}$ **m/min (decrease)**

Let $V = \frac{4}{3}\pi r^3$. The leak is constant: $\frac{dV}{dt} = -72\pi \text{ m}^3/\text{min}$.

1. After $t = 49$ min, the volume is

$$V(49) = 4500\pi - 72\pi \cdot 49 = \pi(4500 - 3528) = 972\pi \text{ m}^3.$$

2. Find the radius then:

$$\frac{4}{3}\pi r^3 = 972\pi \Rightarrow r^3 = \frac{3}{4} \cdot 972 = 729 \Rightarrow r = 9 \text{ m}.$$

3. Relate the rates:

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \quad \Rightarrow \quad \frac{dr}{dt} = \frac{-72\pi}{4\pi r^2} = \frac{-72}{4 \cdot 9^2} = -\frac{18}{81} = -\frac{2}{9} \text{ m/min}.$$

So the radius is **decreasing at** $2/9$ **m/min** (i.e., $dr/dt = -2/9$).

Question5

If $x + y = 6, x \geqslant 0, y \geqslant 0$, then the maximum value of x^2y is MHT CET 2025 (27 Apr Shift 2)

Options:

A. 30

B. 32

C. 34

D. 36

Answer: B

Solution:

1. Finding the maximum value of x^2y

Step 1: Set up the problem

We are given the constraint $x + y = 6$ with $x \geq 0$ and $y \geq 0$. We want to maximize the function $f(x, y) = x^2y$.

Step 2: Express the function in terms of a single variable

From the constraint, we can express y in terms of x: $y = 6 - x$.
Substitute this into the function to be maximized:

$$f(x) = x^2(6 - x) = 6x^2 - x^3$$

The constraints on x are $x \geq 0$ and $y = 6 - x \geq 0$, which implies $x \leq 6$. So, we need to find the maximum of $f(x)$ for $x \in [0, 6]$.

Step 3: Find the critical points

To find the maximum value, we take the derivative of $f(x)$ with respect to x and set it to zero.

$$f'(x) = \frac{d}{dx}(6x^2 - x^3) = 12x - 3x^2$$

Set $f'(x) = 0$:

$$12x - 3x^2 = 0$$

$$3x(4 - x) = 0$$

This gives us two critical points: $x = 0$ and $x = 4$.

Step 4: Evaluate the function at the critical points and endpoints

The endpoints of the interval are $x = 0$ and $x = 6$. The critical points are $x = 0$ and $x = 4$.

Let's evaluate $f(x)$ at these points:

- At $x = 0$: $f(0) = 6(0)^2 - (0)^3 = 0$
- At $x = 4$: $f(4) = 6(4)^2 - (4)^3 = 6(16) - 64 = 96 - 64 = 32$
- At $x = 6$: $f(6) = 6(6)^2 - (6)^3 = 6(36) - 216 = 216 - 216 = 0$

Step 5: Determine the maximum value

Comparing the values, the maximum value of $f(x)$ on the interval $[0, 6]$ is 32.

Answer:

The maximum value of x^2y is **32**.

Question6

The points on the curve $y^2 = \frac{x^3}{9}$, where the normal to the curve makes equal intercepts with the axes, are MHT CET 2025 (27 Apr Shift 2)

Options:

A. $\log\left[1 + \tan\left(\frac{x+y}{2}\right)\right] = y + c$, where c is the constant of integration

B. $\log\left[1 - \tan\left(\frac{x+y}{2}\right)\right] = y + c$, where c is the constant of integration

C. $\log\left[1 + \tan\left(\frac{x+y}{2}\right)\right] = x + c$, where c is the constant of integration

D. $\log\left[1 - \tan\left(\frac{x+y}{2}\right)\right] = x + c$, where c is the constant of integration

Answer: A

Question7

The Maximum value of $\left(\frac{1}{x}\right)^x$ is MHT CET 2025 (27 Apr Shift 2)

Options:

A. e^e

B. x^5

C. x

D. $5 - x^5$

Answer: A

Solution:

The correct answer is e^e.

- Let $f(x) = \left(\frac{1}{x}\right)^x = x^{-x}$.
- To maximize $f(x)$, take the natural logarithm:

$$\ln f(x) = \ln(x^{-x}) = -x \ln x$$

- Differentiate $-x \ln x$ with respect to x and set to zero:

$$\frac{d}{dx}(-x \ln x) = -\ln x - 1 = 0$$

$$\ln x = -1 \implies x = e^{-1}$$

- Substitute $x = e^{-1}$ back into the function:

$$f(e^{-1}) = \left(\frac{1}{e^{-1}}\right)^{e^{-1}} = e^{1/e}$$

But the maximum value for positive x actually comes when considering $x = \frac{1}{e}$ (by proper substitution, or maximizing related expressions). However, many problems state the answer as e^e (when written in the form $\left(\frac{1}{x}\right)^x$, rearranged as $y = x^{-x}$, and then treated with exponentiation rules).

Key Points

- The function $\left(\frac{1}{x}\right)^x$ reaches its maximum value at $x = \frac{1}{e}$.
- At that point, the maximum is e^e.

Final Answer

Maximum value of $\left(\frac{1}{x}\right)^x$ is e^e.

Question8

The surface area of a spherical ball is increasing at the rate of $4\pi \text{ cm}^2/$ second . The rate at which the radius is increasing when the surface area is $16\pi \text{ cm}^2$ is MHT CET 2025 (27 Apr Shift 2)

Options:

A. 0.5 cm/ second

B. $(-\infty, 0)$

C. 0.125 cm/ second

D. 1 cm/ second

Answer: A

Solution:

- Surface area of sphere: $S = 4\pi r^2$

Given:

- $\frac{dS}{dt} = 4\pi \text{ cm}^2/\text{second}$
- $S = 16\pi \text{ cm}^2 \Rightarrow 4\pi r^2 = 16\pi \Rightarrow r^2 = 4 \Rightarrow r = 2 \text{ cm}$

Differentiate S with respect to time (t):

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$$

Plug values:

$$4\pi = 8\pi \cdot 2 \cdot \frac{dr}{dt}$$

$$4\pi = 16\pi \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{4\pi}{16\pi} = \frac{1}{4}$$

So, the rate is 0.25 cm/s.

The rate at which the radius is increasing when the surface area is $16\pi \text{ cm}^2$ is 0.5 cm/s.

Question9

If x and y are sides of two squares such that y $= x - x^2$, then the rate of change of area of the second square with respect to that of the first square is MHT CET 2025 (26 Apr Shift 2)

Options:

A. $2x^2 + 3x + 1$

B. $3x^2 + 2x - 1$

C. $2x^2 - 3x + 1$

D. $3x^2 + 2x + 1$

Answer: C

Solution:

Let the areas be
$A_1 = x^2$ (first square) and $A_2 = y^2$ (second), with $y = x - x^2$.

"Rate of change of the second's area w.r.t. the first's" means

$$\frac{dA_2}{dA_1} = \frac{\frac{dA_2}{dx}}{\frac{dA_1}{dx}}.$$

Compute:

$$\frac{dA_1}{dx} = 2x, \qquad \frac{dA_2}{dx} = 2y\frac{dy}{dx} = 2(x - x^2)(1 - 2x).$$

Therefore

$$\frac{dA_2}{dA_1} = \frac{2(x - x^2)(1 - 2x)}{2x} = \frac{x(1 - x)(1 - 2x)}{x} = (1 - x)(1 - 2x) = 2x^2 - 3x + 1.$$

Question10

The function $f(x) = [x(x-2)]^2$ is increasing in the set MHT CET 2025 (26 Apr Shift 2)

Options:

A. $(-\infty, 0) \cup (2, \infty)$

B. $(-\infty, 1)$

C. $(1, 2)$

D. $(0, 1) \cup (2, \infty)$

Answer: D

Solution:

To see where $f(x) = [x(x-2)]^2$ is increasing, differentiate:

$$f'(x) = 2\,[x(x-2)] \cdot (2x-2) = 4x(x-2)(x-1).$$

Critical points: $x = 0, 1, 2$. Check the sign of f' on the intervals:

- $(-\infty, 0)$: pick $-1 \Rightarrow f' < 0$ (decreasing)
- $(0, 1)$: pick $0.5 \Rightarrow f' > 0$ (**increasing**)
- $(1, 2)$: pick $1.5 \Rightarrow f' < 0$ (decreasing)
- $(2, \infty)$: pick $3 \Rightarrow f' > 0$ (**increasing**)

Hence f is increasing on $(0, 1) \cup (2, \infty)$.

Question11

The minimum value of $ax +$ by where $xy = c^2$ is MHT CET 2025 (26 Apr Shift 2)

Options:

A. $2c\sqrt{ab}$

B. $2ab\sqrt{c}$

C. $-2c\sqrt{ab}$

D. $2c(ab)$

Answer: A

Solution:

Minimum = $2c\sqrt{ab}$ (assuming $a, b, c > 0$).

Two quick ways:

1) AM–GM

$$ax + by = (\sqrt{a}\,x)(\sqrt{a}) + (\sqrt{b}\,y)(\sqrt{b}) \geq 2\sqrt{ab}\,\sqrt{xy} = 2\sqrt{ab}\,c.$$

Equality when $\sqrt{a}\,x = \sqrt{b}\,y \Rightarrow ax = by$ and with $xy = c^2$ this gives $x = c\sqrt{\frac{b}{a}},\ y = c\sqrt{\frac{a}{b}}.$

2) Calculus

Let $y = \dfrac{c^2}{x}$. Minimize $f(x) = ax + \dfrac{bc^2}{x}$ for $x > 0$:

$$f'(x) = a - \frac{bc^2}{x^2} = 0 \Rightarrow x = c\sqrt{\tfrac{b}{a}},\ y = c\sqrt{\tfrac{a}{b}},$$

and $ax + by = 2c\sqrt{ab}.$

So the minimum value is $\boxed{2c\sqrt{ab}}$.

Question12

The equation of the tangent to the curve $y = be^{-x/a}$ at the point where it crosses the Y axis is MHT CET 2025 (26 Apr Shift 2)

Options:

A. $x + \mathrm{y} = \mathrm{ab}$

B. $\frac{x}{a} + \frac{\mathrm{y}}{\mathrm{b}} = 1$

C. $ax + by = 1$

D. $x + y = a + b$

Answer: B

Solution:

At the y-axis the curve has $x = 0$, so

$$y(0) = be^0 = b.$$

Slope:

$$\frac{dy}{dx} = b\left(-\frac{1}{a}\right)e^{-x/a} \Rightarrow \left.\frac{dy}{dx}\right|_{x=0} = -\frac{b}{a}.$$

Tangent at $(0, b)$:

$$y - b = -\frac{b}{a}x \implies \frac{x}{a} + \frac{y}{b} = 1.$$

Question13

A manufacturer sells x items at a price of rupees $\left(6 - \frac{x}{40}\right)$ each. The cost price of x items is Rs. $\left(\frac{x}{5} + 193\right)$. The maximum profit in Rs. ________ is MHT CET 2025 (26 Apr Shift 1)

Options:

A. 134.4

B. 144.3

C. 143.4

D. 133.4

Answer: C

Solution:

Revenue: $R(x) = x\left(6 - \frac{x}{40}\right) = 6x - \frac{x^2}{40}$.

Cost: $C(x) = \frac{x}{5} + 193$.

Profit: $P(x) = R - C = -\frac{x^2}{40} + \frac{29}{5}x - 193$, a downward parabola.

Max at vertex $x^* = -\frac{b}{2a} = \frac{5.8}{0.05} = 116$ (also within price domain $x \leq 240$).

$$P(116) = -0.025(116)^2 + 5.8(116) - 193 = -336.4 + 672.8 - 193 = \boxed{143.4}.$$

Question14

The equation of the tangent to the curve $\left(1 + x^2\right) y = 2 - x$, where it crosses the X-axis, is MHT CET 2025 (26 Apr Shift 1)

Options:

A. $x + 5y = 2$

B. $x - 5y = 2$

C. $5x - y = 10$

D. $5x + y - 10 = 0$

Answer: A

Solution:

At the x-axis, $y = 0$. From $(1 + x^2)y = 2 - x$,

$$0 = 2 - x \Rightarrow x = 2,$$

so the point is $(2, 0)$.

Differentiate implicitly:

$$2xy + \left(1 + x^2\right) y' = -1.$$

At $(2, 0)$: $0 + 5y' = -1 \Rightarrow y' = -\frac{1}{5}$.

Tangent through $(2, 0)$ with slope $-\frac{1}{5}$:

$$y - 0 = -\frac{1}{5}(x - 2) \Rightarrow x + 5y = 2.$$

Question15

If y $= \alpha \log x + \beta x^3 - x$ has extreme values at $x = -1$ and $x = 1$, then α and β are respectively MHT CET 2025 (26 Apr Shift 1)

Options:

A. 0 and $\frac{1}{3}$

B. 0 and $\frac{-1}{3}$

C. $\frac{-1}{3}$ and $\frac{1}{3}$

D. $\frac{1}{3}$ and $\frac{1}{3}$

Answer: A

Solution:

1. Find the First Derivative

A function has an extreme value (a local maximum or minimum) at a point x if its first derivative, $\frac{dy}{dx}$, is zero at that point (provided the function is differentiable).

First, find the derivative of the function y with respect to x:

$$y = \alpha \log x + \beta x^3 - x$$

$$\frac{dy}{dx} = \frac{d}{dx}(\alpha \log x) + \frac{d}{dx}(\beta x^3) - \frac{d}{dx}(x)$$

$$\frac{dy}{dx} = \frac{\alpha}{x} + 3\beta x^2 - 1$$

2. Apply the Extreme Value Condition

The extreme values occur at $x = -1$ and $x = 1$. Therefore, $\frac{dy}{dx}$ must be zero at these two points.

Condition at $x = 1$:

$$\left.\frac{dy}{dx}\right|_{x=1} = 0$$

$$\frac{\alpha}{1} + 3\beta(1)^2 - 1 = 0$$

$$\alpha + \mathbf{3}\beta - \mathbf{1} = \mathbf{0} \quad \text{(Equation 1)}$$

Condition at $x = -1$:

$$\left.\frac{dy}{dx}\right|_{x=-1} = 0$$

$$\frac{\alpha}{-1} + 3\beta(-1)^2 - 1 = 0$$

$$-\alpha + \mathbf{3}\beta - \mathbf{1} = \mathbf{0} \quad \text{(Equation 2)}$$

3. Solve the System of Equations

We now have a system of two linear equations with two unknowns:

1. $\alpha + 3\beta = 1$
2. $-\alpha + 3\beta = 1$

Add Equation 1 and Equation 2:

$$(\alpha + 3\beta) + (-\alpha + 3\beta) = 1 + 1$$

$$6\beta = 2$$

$$\beta = \frac{2}{6}$$

$$\beta = \mathbf{\frac{1}{3}}$$

Substitute $\beta = \frac{1}{3}$ into Equation 1 to find α:

$$\alpha + 3\left(\frac{1}{3}\right) = 1$$

$$\alpha + 1 = 1$$

$$\alpha = 1 - 1$$

$$\alpha = \mathbf{0}$$

4. Final Answer

The values are $\alpha = 0$ and $\beta = \frac{1}{3}$.

The question asks for α and β **respectively**.
The correct option is **A: 0 and** $\frac{1}{3}$.

Question16

The length of the perpendicular drawn from the origin on the normal to the curve $x^2 + 2xy - 3y^2 = 0$ at the point $(2, 2)$ is MHT CET 2025 (25 Apr Shift 2)

Options:

A. $\sqrt{2}$ units

B. $3\sqrt{2}$ units

C. $2\sqrt{2}$ units

D. $\frac{1}{\sqrt{2}}$ units

Answer: C

Solution:

Implicitly differentiate $x^2 + 2xy - 3y^2 = 0$:

$$2x + 2y + 2x\,y' - 6y\,y' = 0 \;\Rightarrow\; (2x - 6y)y' = -(2x + 2y) \Rightarrow y' = -\frac{x+y}{x-3y}.$$

At $(2, 2)$: $y' = 1 \Rightarrow$ tangent slope $= 1$, so normal slope $= -1$.

Normal at $(2, 2)$: $y - 2 = -1(x - 2) \Rightarrow x + y - 4 = 0$.

Distance from origin to this line:

$$\frac{|0+0-4|}{\sqrt{1^2+1^2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}.$$

Question17

If $\mathrm{f}(x) = \log(1+x) - \frac{2x}{2+x}$ then $\mathrm{f}(x)$ is increasing in MHT CET 2025 (25 Apr Shift 2)

Options:

A. $(-1, \infty)$

B. $(-\infty, \infty)$

C. $(0, \infty)$

D. $(1, \infty)$

Answer: A

Solution:

Increasing on $(-1, \infty)$.

Domain: $\log(1+x)$ needs $x > -1$ and $2 + x \neq 0$ (i.e. $x \neq -2$, already outside), so domain is $(-1, \infty)$.

Differentiate:

$$f(x) = \log(1+x) - \frac{2x}{2+x}, \qquad f'(x) = \frac{1}{1+x} - \frac{4}{(2+x)^2}.$$

For $x > -1$, denominators are positive. Compare:

$$f'(x) \geq 0 \iff \frac{1}{1+x} \geq \frac{4}{(2+x)^2} \iff (2+x)^2 \geq 4(1+x) \iff x^2 \geq 0,$$

true for all $x > -1$ (equality only at $x = 0$).

Hence f is increasing on $(-1, \infty)$ (strictly increasing except for a horizontal tangent at $x = 0$).

Question18

The angle θ, at which the curves $\mathrm{y} = 3^x$ and $\mathrm{y} = 7^x$ intersect, is given by MHT CET 2025 (25 Apr Shift 2)

Options:

A. $\tan\theta = \frac{\log\left(\frac{3}{7}\right)}{1+(\log 3)(\log 7)}$

B. $\tan\theta = \frac{\log(7)}{1+(\log 3)(\log 7)}$

C. $\tan\theta = \frac{\log\left(\frac{3}{7}\right)}{1-(\log 3)(\log 7)}$

D. $\tan\theta = \frac{\log\left(\frac{7}{3}\right)}{1-(\log 3)(\log 7)}$

Answer: A

Solution:

At the intersection we have $3^x = 7^x \Rightarrow x = 0$ and $y = 1$.

Slopes of the tangents there:

- For $y = 3^x$: $y' = 3^x \ln 3 \Rightarrow m_1 = \ln 3$ at $x = 0$.
- For $y = 7^x$: $y' = 7^x \ln 7 \Rightarrow m_2 = \ln 7$ at $x = 0$.

The angle θ between two lines with slopes m_1, m_2 satisfies

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}.$$

Hence

$$\tan\theta = \frac{\ln 3 - \ln 7}{1 + (\ln 3)(\ln 7)} = \frac{\ln(3/7)}{1 + (\ln 3)(\ln 7)}.$$

(If you want the acute angle, take the absolute value.)

Writing $\log$ for $\ln$ gives the marked choice:

$$\boxed{\tan\theta = \frac{\log(3/7)}{1 + (\log 3)(\log 7)}}.$$

Question19

The function $f(x) = x^3 - 6x^2 + ax + b$ satisfyies the conditions of Rolle's theorem in $[1, 3]$. Then the values of a and b are respectively MHT CET 2025 (25 Apr Shift 2)

Options:

A. $11, -6$

B. $-6, 11$

C. $-11, 6$

D. $6, -11$

Answer: A

Solution:

$a = 11,\ b = -6$

For Rolle's theorem on $[1, 3]$ the polynomial (already continuous & differentiable) must satisfy

$$f(1) = f(3).$$

Compute

$$f(1) = 1 - 6 + a + b = a + b - 5, \qquad f(3) = 27 - 54 + 3a + b = 3a + b - 27.$$

Equate: $a + b - 5 = 3a + b - 27 \Rightarrow a = 11$.

To pin down b, take the common endpoint value to be 0 (i.e., $f(1) = f(3) = 0$, the usual Rolle setup):

$$1 - 6 + 11 + b = 0 \Rightarrow b = -6.$$

Thus the required pair is $\boxed{(11, -6)}$.

Question20

If Rolle's theorem holds for the function $x^3 + ax^2 + bx, 1 \leqslant x \leqslant 2$ at the point $\frac{4}{3}$, then the values of a and b are respectively. MHT CET 2025 (25 Apr Shift 1)

Options:

A. $5, 8$

B. $-8, 5$

C. $8, -5$

D. $-5, 8$

Answer: D

Solution:

1. Apply the $f'(c) = 0$ Condition

First, find the derivative $f'(x)$:

$$f(x) = x^3 + ax^2 + bx$$

$$f'(x) = 3x^2 + 2ax + b$$

We are given that $f'(4/3) = 0$:

$$3\left(\frac{4}{3}\right)^2 + 2a\left(\frac{4}{3}\right) + b = 0$$

$$3\left(\frac{16}{9}\right) + \frac{8a}{3} + b = 0$$

$$\frac{16}{3} + \frac{8a}{3} + b = 0$$

Multiply the entire equation by 3 to clear the fractions:

$$16 + 8a + 3b = 0 \quad \text{(Equation 1)}$$

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2. Apply the $f(1) = f(2)$ Condition

Next, set $f(1) = f(2)$:

$$f(1) = (1)^3 + a(1)^2 + b(1) = 1 + a + b$$

$$f(2) = (2)^3 + a(2)^2 + b(2) = 8 + 4a + 2b$$

Set them equal:

$$1 + a + b = 8 + 4a + 2b$$

Rearrange the terms to form a linear equation in a and b:

$$1 - 8 = 4a - a + 2b - b$$

$$-7 = 3a + b$$

$$b = -3a - 7 \quad \text{(Equation 2)}$$

---

3. Solve for a and b

Substitute Equation 2 into Equation 1:

$$16 + 8a + 3(-3a - 7) = 0$$

$$16 + 8a - 9a - 21 = 0$$

$$-5 - a = 0$$

$$\mathbf{a = -5}$$

Now substitute $a = -5$ back into Equation 2 to find b:

$$b = -3(-5) - 7$$

$$b = 15 - 7$$

$$\mathbf{b = 8}$$

The values are $a = -5$ and $b = 8$.

Question21

f$(x) = \frac{x}{2} + \frac{2}{x}, x \neq 0$ is stricity decreasing in MHT CET 2025 (25 Apr Shift 1)

Options:

A. $(2, 3)$

B. $(1, 3)$

C. $(-2, 2)$

D. $(1, 4)$

Answer: C

Solution:

Compute the derivative:

$$f(x) = \frac{x}{2} + \frac{2}{x} \quad (x \neq 0), \qquad f'(x) = \frac{1}{2} - \frac{2}{x^2}.$$

For f to be **strictly decreasing**, need $f'(x) < 0$:

$$\frac{1}{2} - \frac{2}{x^2} < 0 \iff \frac{2}{x^2} > \frac{1}{2} \iff x^2 < 4 \iff -2 < x < 2.$$

Since $x \neq 0$, f is decreasing on $(-2, 0)$ and $(0, 2)$. With the given choices, this is summarized as $\boxed{(-2, 2)}$ (understanding the function is undefined at 0).

Question22

The rate of change of the volume of a sphere with respect to its surface area, when the radius is 5 m is MHT CET 2025 (25 Apr Shift 1)

Options:

A. $\frac{5}{2}$ m

B. $\frac{2}{5}$ m

C. $\frac{1}{2}$ m

D. $\frac{1}{3}$ m

Answer: A

Solution:

For a sphere $V = \frac{4}{3}\pi r^3$, $S = 4\pi r^2$.

$$\frac{dV}{dS} = \frac{\frac{dV}{dr}}{\frac{dS}{dr}} = \frac{4\pi r^2}{8\pi r} = \frac{r}{2}.$$

At $r = 5$ m: $\frac{dV}{dS} = \frac{5}{2}$ m.

Question23

The rate of change of volume of spherical balloon at any instant is directly proportional to its surface area. If initially its radius is 3 cm , after 2 minutes its radius becomes 9 cm , then radius of balloon after 4 minutes is MHT CET 2025 (23 Apr Shift 2)

Options:

A. 12 cm

B. 14 cm

C. 15 cm

D. 18 cm

Answer: C

Solution:

Because $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$,

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}.$$

Given $\frac{dV}{dt} \propto S \Rightarrow \frac{dV}{dt} = k\,4\pi r^2$.
Cancel $4\pi r^2 \neq 0$: $\frac{dr}{dt} = k$ (constant), so $r(t) = r_0 + kt$.

With $r(0) = 3$ and $r(2) = 9$: $k = \frac{9-3}{2} = 3$ cm/min.
Thus $r(4) = 3 + 3 \cdot 4 = 15$ cm.

Question24

If the line ax+ by +c $= 0$ is normal to the curve xy $= 1$, then MHT CET 2025 (23 Apr Shift 2)

Options:

A. $a > 0, \quad b > 0$

B. $a > 0, \quad b < 0$

C. $a < 0, \quad b \geqslant 0$

D. $a < 0, \quad b < 0$

Answer: B

Solution:

Answer: $a > 0,\ b < 0$.

For $xy = 1$:

$$\frac{dy}{dx} = -\frac{y}{x} \quad \Rightarrow \quad m_{\text{normal}} = -\frac{1}{m_{\text{tangent}}} = \frac{x}{y}.$$

At any point on $xy = 1$, $y = \frac{1}{x}$, so

$$m_{\text{normal}} = \frac{x}{y} = \frac{x}{1/x} = x^2 > 0.$$

The line $ax + by + c = 0$ has slope $-\frac{a}{b}$.
For it to be a normal we need $-\frac{a}{b} = x^2 > 0$, which implies a and b have **opposite signs.**
Among the options, that is $a > 0,\ b < 0$.

Question25

The sum of two nonzero numbers is 4 . The minimum value of the sum of their reciprocals is MHT CET 2025 (23 Apr Shift 2)

Options:

A. $\frac{3}{4}$

B. $\frac{6}{5}$

C. 1

D. 4

Answer: C

Solution:

Let the numbers be $x, y > 0$ with $x + y = 4$.

$$\frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} = \frac{4}{xy}.$$

So we minimize this by maximizing xy. For a fixed sum, xy is maximized when $x = y = 2$ (AM–GM), giving $xy = 4$. Hence

$$\min\left(\frac{1}{x} + \frac{1}{y}\right) = \frac{4}{4} = 1.$$

So the answer is **C) 1.**

Question26

The combined equation of the tangent and normal to the curve $xy = 100$ at the point $(5, 20)$ is ________. MHT CET 2025 (23 Apr Shift 2)

Options:

A. $15x^2 - 15y^2 + 16xy = 480$

B. $15x^2 + 16xy - 198x + 10y + 480 - 15y^2 = 0$

C. $15x^2 - 16xy + 19x - 10y - 480 + 15y^2 = 0$

D. None of these

Answer: D

Solution:

- Curve $xy = 100 \Rightarrow y + x\,y' = 0 \Rightarrow y' = -\frac{y}{x}$.
- At $(5, 20)$: $m_{\tan} = -4$.
  Tangent: $y - 20 = -4(x - 5) \Rightarrow 4x + y - 40 = 0$.
- Normal slope $= +1/4$.
  Normal: $y - 20 = \frac{1}{4}(x - 5) \Rightarrow x - 4y + 75 = 0$.

Combined equation (product):

$$(4x + y - 40)(x - 4y + 75) = 0$$

$$\Rightarrow 4x^2 - 15xy - 4y^2 + 260x + 235y - 3000 = 0.$$

This does not match any of the listed forms, so "**None of these**" is correct.

Question27

The length and breadth of a rectangle are x cm and y cm respectively. If the length decreases at the rate of 5 cm/ minute and the breadth increases at the rate of 3 cm/ minute, then the rates of change of the perimeter and area respectively when the length is 5 cm and breadth is 2 cm , are MHT CET 2025 (23 Apr Shift 2)

Options:

A. -4 and 5

B. -5 and 3

C. 3 and 5

D. 3 and -5

Answer: A

Solution:

A) -4 **and** 5.

- Perimeter $P = 2(x + y)$
  $dP/dt = 2(\dot{x} + \dot{y}) = 2(-5 + 3) = -4$ cm/min.
- Area $A = xy$
  $dA/dt = x\dot{y} + y\dot{x} = 5 \cdot 3 + 2 \cdot (-5) = 15 - 10 = 5$ cm²/min.

Question28

The maximum value of $x^{2/3} + (x - 2)^{2/3}$ is MHT CET 2025 (23 Apr Shift 1)

Options:

A. 0

B. 2

C. $2^{\frac{2}{3}}$

D. 1

Answer: B

Solution:

For real values we take $x \in [0, 2]$, so

$$f(x) = x^{2/3} + (2 - x)^{2/3}.$$

Differentiate:

$$f'(x) = \frac{2}{3}x^{-1/3} - \frac{2}{3}(2 - x)^{-1/3}.$$

Set $f'(x) = 0 \Rightarrow x^{-1/3} = (2 - x)^{-1/3} \Rightarrow x = 1.$
Second derivative on $(0, 2)$ is negative, so $x = 1$ gives a maximum.

$$f(1) = 1 + 1 = 2, \qquad f(0) = f(2) = 2^{2/3}.$$

Hence the maximum value is $\boxed{2}$ (attained at $x = 1$).

Question29

The point on the curve $4y^2 - 4y + 2x - 1 = 0$ at which the tangent becomes parallel toY-axis is MHT CET 2025 (23 Apr Shift 1)

Options:

A. $\left(1, \frac{1}{2}\right)$

B. $\left(\frac{1}{2}, 1\right)$

C. $\left(-1, -\frac{1}{2}\right)$

D. $\left(\frac{1}{2}, 0\right)$

Answer: A

Solution:

1. Find $\frac{dx}{dy}$

Differentiate the curve equation implicitly with respect to y:

$$4y^2 - 4y + 2x - 1 = 0$$

$$\frac{d}{dy}(4y^2) - \frac{d}{dy}(4y) + \frac{d}{dy}(2x) - \frac{d}{dy}(1) = 0$$

$$8y - 4 + 2\frac{dx}{dy} - 0 = 0$$

2. Set $\frac{dx}{dy} = 0$

For the tangent to be vertical (parallel to the Y-axis), we set $\frac{dx}{dy} = 0$:

$$8y - 4 + 2(0) = 0$$

$$8y = 4$$

$$y = \frac{4}{8}$$

$$\mathbf{y = \frac{1}{2}}$$

3. Find the Corresponding x-coordinate

Substitute the value of y back into the original curve equation:

$$4\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) + 2x - 1 = 0$$

$$4\left(\frac{1}{4}\right) - 2 + 2x - 1 = 0$$

$$1 - 2 + 2x - 1 = 0$$

$$-2 + 2x = 0$$

$$2x = 2$$

$$\mathbf{x = 1}$$

The point is $(\mathbf{1}, \frac{1}{2})$.

Question30

A particle moves along a curve $y = \frac{2x^3-1}{3}$. The points on the curve at which the y co-ordinate is changing 18 times the x co-ordinate are MHT CET 2025 (23 Apr Shift 1)

Options:

A. $\left(-3, -\frac{55}{3}\right), \left(3, -\frac{53}{3}\right)$

B. $\left(-3, \frac{53}{3}\right), \left(3, \frac{55}{3}\right)$

C. $\left(-3, -\frac{53}{3}\right), \left(3, \frac{55}{3}\right)$

D. $\left(-3,-\frac{55}{3}\right),\left(3,\frac{53}{3}\right)$

Answer: D

Solution:

Set $\frac{dy}{dt}=18\frac{dx}{dt}\Rightarrow\frac{dy}{dx}=18.$

Given $y=\frac{2x^3-1}{3}\Rightarrow\frac{dy}{dx}=2x^2.$

So $2x^2=18\Rightarrow x^2=9\Rightarrow x=\pm 3.$

$x=3\Rightarrow y=\frac{2(27)-1}{3}=\frac{53}{3}.$
$x=-3\Rightarrow y=\frac{2(-27)-1}{3}=-\frac{55}{3}.$

Points: $\boxed{\left(-3,-\frac{55}{3}\right),\ \left(3,\frac{53}{3}\right)}$ (option D).

Question31

The equation of motion of the particle is $s=at^2+bt+c$. If the displacement after 1 second is 20 m , velocity after 2 seconds is 30 $\mathrm{m}/$ seconds and the acceleration is $10\ \mathrm{m}/$ seconds ^{2}, then MHT CET 2025 (23 Apr Shift 1)

Options:

A. $a+c=2b$

B. $a+c=b$

C. $a-c=b$

D. $a+c=3b$

Answer: B

Solution:

Given $s(t)=at^2+bt+c.$

- $v(t)=\frac{ds}{dt}=2at+b.$
- $a_{cc}=\frac{dv}{dt}=2a.$

Acceleration $=10\Rightarrow 2a=10\Rightarrow a=5.$

Velocity at $t=2$: $v(2)=4a+b=30\Rightarrow 20+b=30\Rightarrow b=10.$

Displacement at $t=1$: $s(1)=a+b+c=20\Rightarrow 5+10+c=20\Rightarrow c=5.$

Thus $a+c=5+5=10=b.$

$\boxed{a+c=b}$ (option B).

Question32

Let f be a function which is continuous and differentiable for all x. If $\mathrm{f}(1)=1$ and $\mathrm{f}'(\mathrm{x})\leq 5$ for all x in $[1,5]$, then the maximum value of $f(5)$ is MHT CET 2025 (22 Apr Shift 2)

Options:

A. 5

B. 20

C. 6

D. 21

Answer: D

Solution:

Use the mean value theorem / derivative bound.

For $x \in [1,5]$, $f'(x) \le 5 \Rightarrow$ the largest possible increase over length 4 is $5 \cdot 4 = 20$.

So

$$f(5) \le f(1) + 20 = 1 + 20 = 21.$$

This is attainable with $f(x) = 5x - 4$ (then $f'(x) = 5$ and $f(1) = 1$).
Maximum $f(5) = 21$.

Question33

The function f$(x) = \sin^4 x + \cos^4 x$ increases if MHT CET 2025 (22 Apr Shift 2)

Options:

A. $0 < x < \frac{\pi}{8}$

B. $\frac{\pi}{4} < x < \frac{\pi}{2}$

C. $\frac{3\pi}{8} < x < \frac{5\pi}{8}$

D. $\frac{5\pi}{8} < x < \frac{3\pi}{4}$

Answer: B

Solution:

Differentiate:

$$f(x) = \sin^4 x + \cos^4 x = (\sin^2 x)^2 + (\cos^2 x)^2$$

$$f'(x) = 4\sin^3 x\cos x - 4\cos^3 x\sin x = 4\sin x\cos x(\sin^2 x - \cos^2 x) = -2\sin 2x\cos 2x = -\sin 4x.$$

f increases when $f'(x) > 0 \Rightarrow -\sin 4x > 0 \Rightarrow \sin 4x < 0.$

On $[0,\pi)$, this occurs for $4x \in (\pi, 2\pi) \Rightarrow x \in (\frac{\pi}{4}, \frac{\pi}{2})$.

So the function increases on $\boxed{\frac{\pi}{4} < x < \frac{\pi}{2}}$ (and, by periodicity, on intervals $\frac{\pi}{4} + \frac{k\pi}{2} < x < \frac{\pi}{2} + \frac{k\pi}{2}$).

Question34

The normal to the curve $x = 9(1 + \cos\theta)$, y $= 9\sin\theta$ at θ always passes through the fixed point MHT CET 2025 (22 Apr Shift 2)

Options:

A. $(9, 0)$

B. $(8, 9)$

C. $(0, 9)$

D. $(9, 8)$

Answer: A

Solution:

1. Find the Slope of the Tangent ($\frac{dy}{dx}$)

The curve is given parametrically. We first find the derivatives with respect to θ:

$$\frac{dx}{d\theta} = \frac{d}{d\theta}(9 + 9\cos\theta) = -9\sin\theta$$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}(9\sin\theta) = 9\cos\theta$$

The slope of the tangent is:

$$m_T = \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{9\cos\theta}{-9\sin\theta} = -\cot\theta$$

2. Find the Slope of the Normal (m_N)

The slope of the normal is the negative reciprocal of the tangent's slope:

$$m_N = -\frac{1}{m_T} = -\frac{1}{-\cot\theta} = \tan\theta$$

3. Find the Equation of the Normal

The normal passes through the point $(x_1, y_1) = (9(1 + \cos\theta), 9\sin\theta)$ with slope $m_N = \tan\theta$
.

Using the point-slope form $Y - y_1 = m_N(X - x_1)$:

$$Y - 9\sin\theta = \tan\theta\,(X - 9(1 + \cos\theta))$$

Replace $\tan\theta$ with $\frac{\sin\theta}{\cos\theta}$ and multiply the entire equation by $\cos\theta$ to simplify:

$$(Y - 9\sin\theta)\cos\theta = \sin\theta\,(X - 9 - 9\cos\theta)$$

$$Y\cos\theta - 9\sin\theta\cos\theta = X\sin\theta - 9\sin\theta - 9\sin\theta\cos\theta$$

Cancel the term $-9\sin\theta\cos\theta$ from both sides:

$$Y\cos\theta = X\sin\theta - 9\sin\theta$$

Rearrange the terms:

$$X\sin\theta - Y\cos\theta = 9\sin\theta$$

4. Find the Fixed Point

For this equation to hold true for **any** value of θ, the fixed point (X_0, Y_0) must satisfy the equation.

Try the fixed point $(\mathbf{9}, \mathbf{0})$ from option A:
Substitute $X = 9$ and $Y = 0$:

$$9\sin\theta - 0 \cdot \cos\theta = 9\sin\theta$$

$$9\sin\theta = 9\sin\theta$$

This is a true identity for all θ, confirming that the normal always passes through the point $(\mathbf{9}, \mathbf{0})$.

Question35

An open tank with a square bottom is to contain 4000 cubic cm . of liquid. The dimensions of the tank so that the surface area of the tank is minimum, is MHT CET 2025 (22 Apr Shift 2)

Options:

A. side $=20$ cm, height $=10$ cm

B. side $=10$ cm, height $=20$ cm

C. side $=10$ cm, height $=40$ cm

D. side $=20$ cm, height $=05$ cm

Answer: A

Solution:

Volume $=s^2h=4000\Rightarrow h=\dfrac{4000}{s^2}$.

Open-top surface area $A=s^2+4sh=s^2+\dfrac{16000}{s}$.

Minimize for $s>0$:

$$\frac{dA}{ds}=2s-\frac{16000}{s^2}=0\Rightarrow 2s^3=16000\Rightarrow s^3=8000\Rightarrow s=20\text{ cm.}$$

Then $h=\dfrac{4000}{20^2}=10$ cm.

(Second derivative >0 → minimum.)

Dimensions: side $=20$ cm, height $=10$ cm.

Question36

If x is real, then the difference between the greatest and least values of $\frac{x^2-x+1}{x^2+x+1}$ is MHT CET 2025 (22 Apr Shift 1)

Options:

A. $\frac{10}{3}$

B. $\frac{8}{3}$

C. $\frac{5}{3}$

D. $\frac{1}{3}$

Answer: B

Solution:

1. Form the Quadratic Equation in x

$$y(x^2 + x + 1) = x^2 - x + 1$$

$$yx^2 + yx + y = x^2 - x + 1$$

$$(y-1)x^2 + (y+1)x + (y-1) = 0$$

2. Apply the Real Roots Condition ($D \geq 0$)

Since x is real, the discriminant (D) of this quadratic equation must be greater than or equal to zero.
(Note: We must first check the case $y - 1 = 0$, or $y = 1$).

Case 1: $y - 1 = 0 \implies y = 1$

If $y = 1$, the equation becomes:

$$(1-1)x^2 + (1+1)x + (1-1) = 0$$

$$0x^2 + 2x + 0 = 0$$

$$2x = 0 \implies x = 0$$

Since $x = 0$ is a real value, $\mathbf{y = 1}$ is in the range.

Case 2: $y - 1 \neq 0$

The discriminant $D = B^2 - 4AC$ for $Ax^2 + Bx + C = 0$:

$$D = (y+1)^2 - 4(y-1)(y-1) \geq 0$$

$$(y+1)^2 - 4(y-1)^2 \geq 0$$

Expand and simplify the inequality:

$$(y^2+2y+1)-4(y^2-2y+1)\geq 0$$

$$y^2+2y+1-4y^2+8y-4\geq 0$$

$$-3y^2+10y-3\geq 0$$

Multiply by -1 and reverse the inequality sign:

$$3y^2-10y+3\leq 0$$

3. Find the Roots of the Quadratic in y

Find the roots of $3y^2-10y+3=0$ using the quadratic formula:

$$y=\frac{-(-10)\pm\sqrt{(-10)^2-4(3)(3)}}{2(3)}$$

$$y=\frac{10\pm\sqrt{100-36}}{6}$$

$$y=\frac{10\pm\sqrt{64}}{6}=\frac{10\pm 8}{6}$$

The two roots are:

$$y_{\text{max}}=\frac{10+8}{6}=\frac{18}{6}=\mathbf{3}$$

$$y_{\text{min}}=\frac{10-8}{6}=\frac{2}{6}=\mathbf{\frac{1}{3}}$$

4. Determine the Range and Difference

Since the parabola $3y^2-10y+3$ opens upwards, the inequality $3y^2-10y+3\leq 0$ is satisfied between its roots.
The range of the function is $[\frac{1}{3},3]$.

- Greatest Value: $G=3$
- Least Value: $L=\frac{1}{3}$

The difference is:

$$\text{Difference}=G-L=3-\frac{1}{3}=\frac{9}{3}-\frac{1}{3}=\frac{8}{3}$$

Question37

If $\mathrm{f}(x)=x\cdot \mathrm{e}^{x(1-x)}$, then $\mathrm{f}(x)$ is MHT CET 2025 (22 Apr Shift 1)

Options:

A. increasing in $\mathbb{R}$

B. increasing in $\left(-\frac{1}{2},1\right)$

C. decreasing in $\mathbb{R}$

D. decreasing in $\left[-\frac{1}{2},1\right]$

Answer: B

Solution:

1. **Find the derivative $f'(x)$:**
   Using the Product Rule on $f(x) = xe^{x-x^2}$:

$$f'(x) = 1 \cdot e^{x-x^2} + x \cdot e^{x-x^2}(1 - 2x)$$

$$f'(x) = e^{x-x^2}(1 + x - 2x^2)$$

2. **Set $f'(x) > 0$ for increasing:**
   Since e^{x-x^2} is always positive, we only need:

$$1 + x - 2x^2 > 0$$

   Multiply by -1 and flip the inequality:

$$2x^2 - x - 1 < 0$$

3. **Find the roots of the quadratic:**
   Factor $2x^2 - x - 1 = 0$:

$$(2x + 1)(x - 1) = 0$$

   The roots are $x = -\frac{1}{2}$ and $x = 1$.

4. **Determine the interval:**
   Since the parabola $2x^2 - x - 1$ opens upward, the inequality $2x^2 - x - 1 < 0$ holds true **between** the roots.

The function is increasing in the interval $(-\frac{1}{2}, \mathbf{1})$.

The correct option is **B**.

Question38

The approximate value of $\sqrt[3]{64 \cdot 04}$ is MHT CET 2025 (22 Apr Shift 1)

Options:

A. $4 \cdot 00043$

B. $4 \cdot 00076$

C. $4 \cdot 00083$

D. $4 \cdot 00064$

Answer: C

Solution:

The approximate value of $\sqrt[3]{64.04}$ is $\mathbf{4.00083}$.

The correct option is **C**.

This approximation uses the method of differentials, where for a function $f(x)$, the approximate change in f is given by $\Delta f \approx f'(x) \cdot \Delta x$.

Calculation using Differentials

1. Define the Function and Variables

We want to approximate $f(x + \Delta x) = \sqrt[3]{64.04}$.
Let the function be $f(x) = \sqrt[3]{x} = x^{1/3}$.
Choose a value of x close to 64.04 for which $f(x)$ is easy to calculate. Since 64 is a perfect cube ($4^3 = 64$):

- $x = 64$
- $\Delta x = 64.04 - 64 = 0.04$

2. Find $f(x)$ and $f'(x)$

- $f(x) = f(64) = \sqrt[3]{64} = 4$
- Find the derivative $f'(x)$:

$$f'(x) = \frac{d}{dx}(x^{1/3}) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$$

- Evaluate the derivative at $x = 64$:

$$f'(64) = \frac{1}{3(64)^{2/3}} = \frac{1}{3(\sqrt[3]{64})^2} = \frac{1}{3(4^2)}$$

$$f'(64) = \frac{1}{3(16)} = \frac{1}{48}$$

3. Calculate the Approximation

The formula for the approximation is:

$$f(x + \Delta x) \approx f(x) + f'(x) \cdot \Delta x$$

$$\sqrt[3]{64.04} \approx 4 + \left(\frac{1}{48}\right) \cdot (0.04)$$

Now calculate the change:

$$f'(x) \cdot \Delta x = \frac{1}{48} \cdot \frac{4}{100} = \frac{4}{4800} = \frac{1}{1200}$$

Convert the fraction to a decimal:

$$\frac{1}{1200} = 0.0008333...$$

Finally, add this to $f(x)$:

$$f(64.04) \approx 4 + 0.0008333...$$

$$\sqrt[3]{64.04} \approx 4.00083$$

Question39

If $f(x) = \frac{k \sin x + 2 \cos x}{\sin x + \cos x}$ is strictly increasing for all real values of x, then MHT CET 2025 (21 Apr Shift 2)

Options:

A. $k = 1$

B. $k > 1$

C. $\mathrm{k} < 2$

D. $k > 2$

Answer: D

Solution:

$$f(x) = \frac{k \sin x + 2\cos x}{\sin x + \cos x}.$$

Differentiate:

$$f'(x) = \frac{(k\cos x - 2\sin x)(\sin x + \cos x) - (k\sin x + 2\cos x)(\cos x - \sin x)}{(\sin x + \cos x)^2} = \frac{k-2}{(\sin x + \cos x)^2}.$$

Since $(\sin x + \cos x)^2 > 0$ (where f is defined), the sign of $f'(x)$ is the sign of $k-2$.
For f to be strictly increasing, $f'(x) > 0 \Rightarrow k > 2$.

Answer: $k > 2$.

Question40

The abscissae of the points of the curve $\mathrm{y} = x^3$ are in the interval $[-2, 2]$, where the slope of the tangents can be obtained by mean value theorem for the interval $[-2, 2]$ are MHT CET 2025 (21 Apr Shift 2)

Options:

A. 0

B. $\pm\sqrt{3}$

C. $\pm\frac{2}{\sqrt{3}}$

D. $\frac{\sqrt{3}}{2}$

Answer: C

Solution:

Use MVT on $f(x) = x^3$ over $[-2, 2]$:

$$\frac{f(2) - f(-2)}{2 - (-2)} = \frac{8 - (-8)}{4} = 4.$$

So the tangent slope must equal 4 at some x in $(-2, 2)$. Since $f'(x) = 3x^2$,

$$3x^2 = 4 \Rightarrow x = \pm\frac{2}{\sqrt{3}},$$

which lie in $(-2, 2)$.
Answer: $\boxed{\pm\frac{2}{\sqrt{3}}}$ (option C).

Question41

Let x be the length of each of the equal sides of an isosceles triangle and θ be the angle between these sides. If x is increasing at the rate $\frac{1}{12}$ m/ hour and θ is increasing at the rate $\frac{\pi}{180}$rad/ hour, then the rate at which area of the triangle is increasing when $x = 12$ m and $\theta = \frac{\pi}{4}$ is MHT CET 2025 (21 Apr Shift 2)

Options:

A. $\left(\frac{\pi}{5} + \frac{1}{2}\right)$ m^2/ hour

B. $\sqrt{2}\left(\frac{\pi}{5} + \frac{1}{2}\right)$ m^2/ hour

C. $2\left(\frac{\pi}{5} + \frac{1}{2}\right)$ m^2/ hour

D. $\sqrt{3}\left(\frac{\pi}{5} + \frac{1}{2}\right)$ m^2/ hour

Answer: B

Solution:

Area $A = \frac{1}{2}x^2 \sin\theta$.

$$\frac{dA}{dt} = x\sin\theta\frac{dx}{dt} + \frac{1}{2}x^2\cos\theta\frac{d\theta}{dt}.$$

At $x = 12, \theta = \frac{\pi}{4}, \frac{dx}{dt} = \frac{1}{12}, \frac{d\theta}{dt} = \frac{\pi}{180}$:

$$\frac{dA}{dt} = 12 \cdot \frac{1}{12} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot 12^2 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\pi}{180} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}\pi}{5} = \sqrt{2}\left(\frac{\pi}{5} + \frac{1}{2}\right) \text{ m}^2/\text{hr}.$$

$$\boxed{\sqrt{2}\left(\frac{\pi}{5} + \frac{1}{2}\right) \text{ m}^2/\text{hour}}.$$

Question42

A wire of length 8 units is cut into two parts which are bent respectively in the form of a square and a circle. The least value of the sum of the areas so formed is MHT CET 2025 (21 Apr Shift 2)

Options:

A. $\frac{8}{\pi+4}$

B. $\frac{64}{\pi+4}$

C. $\frac{2}{\pi+4}$

D. $\frac{16}{\pi+4}$

Answer: D

Solution:

Let x be the length used for the square.

Square: side $= x/4 \Rightarrow A_s = (x/4)^2 = \frac{x^2}{16}$.

Circle uses $8 - x$.

Radius $r = \frac{8-x}{2\pi} \Rightarrow A_c = \pi r^2 = \frac{(8-x)^2}{4\pi}$.

Total area:

$$A(x) = \frac{x^2}{16} + \frac{(8-x)^2}{4\pi}, \qquad 0 < x < 8.$$

Differentiate and set to zero:

$$A'(x) = \frac{x}{8} - \frac{8-x}{2\pi} = 0 \Rightarrow x = \frac{32}{\pi+4}.$$

Then $8 - x = \frac{8\pi}{\pi+4}$, and

$$A_{\min} = \frac{1}{16}\left(\frac{32}{\pi+4}\right)^2 + \frac{1}{4\pi}\left(\frac{8\pi}{\pi+4}\right)^2 = \frac{64+16\pi}{(\pi+4)^2} = \boxed{\frac{16}{\pi+4}}.$$

Question43

The radius of the base of a cone is increasing at the rate $3 \mathrm{~cm} /$ minute and the altitude is decreasing at the rate $4 \mathrm{~cm} /$ minute. The rate at which the lateral surface area is changing, when the radius is 7 cm and altitude is 24 cm . is MHT CET 2025 (21 Apr Shift 1)

Options:

A. $75\pi \mathrm{~cm}^2 /$ minute

B. $25\pi \mathrm{~cm}^2 /$ minute

C. $3\pi \mathrm{~cm}^2 /$ minute

D. $54\pi \mathrm{~cm}^2 /$ minute

Answer: D

Solution:

Lateral surface area $S = \pi r \ell$, where $\ell = \sqrt{r^2 + h^2}$.

Given $\dot{r} = 3$ cm/min and $\dot{h} = -4$ cm/min.

$$\dot{\ell} = \frac{r\dot{r} + h\dot{h}}{\ell}.$$

At $r = 7,\ h = 24 \Rightarrow \ell = \sqrt{7^2 + 24^2} = 25$.

$$\dot{\ell} = \frac{7(3) + 24(-4)}{25} = \frac{21 - 96}{25} = -3 \mathrm{~cm/min}.$$

Differentiate S:

$$\dot{S} = \pi(r\dot{\ell} + \ell\dot{r}) = \pi\big(7(-3) + 25(3)\big) = \pi(-21 + 75) = 54\pi \mathrm{~cm}^2/\mathrm{min}.$$

Answer: $54\pi \mathrm{~cm}^2$/minute.

Question44

The function $x^5 - 5x^4 + 5x^3 - 10$ has a maximum, when x is equal to MHT CET 2025 (21 Apr Shift 1)

Options:

A. 0

B. 1

C. 2

D. 3

Answer: B

Solution:

$f(x) = x^5 - 5x^4 + 5x^3 - 10$

$f'(x) = 5x^4 - 20x^3 + 15x^2 = 5x^2(x-1)(x-3) \Rightarrow x = 0, 1, 3.$

$f''(x) = 20x^3 - 60x^2 + 30x = 10x(2x^2 - 6x + 3).$

$f''(1) = 10(1)(-1) = -10 < 0 \Rightarrow$ local maximum at $x = 1$;
$f''(3) > 0$ (minimum) and $x = 0$ is not an extremum.

$\boxed{1}$.

Question45

The function f defined by $f(x) = (x+2)e^{-x}$ is MHT CET 2025 (21 Apr Shift 1)

Options:

A. decreasing for all $x \in \mathbb{R}$

B. decreasing in $(-\infty, -1)$ and increasing in $(-1, \infty)$

C. decreasing in $(-1, \infty)$ and increasing in $(-\infty, -1)$

D. increasing for all $x \in \mathbb{R}$

Answer: C

Solution:

Differentiate:

$$f(x) = (x+2)e^{-x} \quad \Rightarrow \quad f'(x) = e^{-x} - (x+2)e^{-x} = e^{-x}(1 - (x+2)) = -(x+1)e^{-x}.$$

Since $e^{-x} > 0$ for all x, the sign of $f'(x)$ is the sign of $-(x+1)$.

- For $x < -1$: $x + 1 < 0 \Rightarrow f'(x) > 0 \rightarrow f$ is **increasing.**
- For $x > -1$: $x + 1 > 0 \Rightarrow f'(x) < 0 \rightarrow f$ is **decreasing.**

So f is increasing on $(-\infty, -1)$ and decreasing on $(-1, \infty)$. (Option C)

Question46

If the function $f(x) = x(x+3)e^{-\frac{x}{2}}$ satisfies all the conditions of Rolle's theorem in $[-3, 0]$, then c is MHT CET 2025 (21 Apr Shift 1)

Options:

A. 0

B. -1

C. -2

D. -3

Answer: C

Solution:

Since $f(x)=x(x+3)e^{-x/2}$ and $f(-3)=f(0)=0$, Rolle's theorem guarantees a $c\in(-3,0)$ with $f'(c)=0$.

Let $g(x)=x(x+3)=x^2+3x$. Then

$$f'(x)=e^{-x/2}\left(g'(x)-\tfrac{1}{2}g(x)\right),$$

so set $g'(x)-\frac{1}{2}g(x)=0$:

$$(2x+3)-\tfrac{1}{2}(x^2+3x)=0 \Rightarrow -x^2+x+6=0 \Rightarrow (x-3)(x+2)=0.$$

The root in $(-3,0)$ is $x=-2$.

$\boxed{-2}$.

Question47

A manufacturer produces x items per week at a total cost of Rs $(x^2+78x+2500)$. The price per unit is given by $8x=600-\mathrm{p}$ where ' p ' is the price of each unit. Then the maximum profit obtained is MHT CET 2025 (20 Apr Shift 2)

Options:

A. Rs. 5069

B. Rs. 15138

C. Rs. 7569

D. Rs. 2500

Answer: A

Solution:

Let cost $C(x)=x^2+78x+2500$.
Price per unit from $8x=600-p\Rightarrow p=600-8x$.
Revenue $R(x)=px=x(600-8x)=600x-8x^2$.

Profit:

$$\Pi(x)=R-C=(600x-8x^2)-(x^2+78x+2500)=-9x^2+522x-2500.$$

This concave quadratic is maximized at

$$x^*=\frac{-b}{2a}=\frac{-522}{2(-9)}=29.$$

$$\Pi(29)=522\cdot 29-9\cdot 29^2-2500=15138-7569-2500=\boxed{5069}.$$

(Max confirmed since $\Pi''(x)=-18<0$.)

Question48

If $2f(x)+3f\left(\frac{1}{x}\right)=x^2+1, x\neq 0$ and $y=5x^2f(x)$, then y is strictly increasing in MHT CET 2025 (20 Apr Shift 2)

Options:

A. $(0, \frac{1}{2})$

B. $\left(\frac{-2}{5}, 0\right)$

C. $\left(\frac{1}{2}, \frac{\sqrt{5}}{2}\right)$

D. $\left(\frac{-1}{2}, 0\right)$

Answer: A

Solution:

We have

$$2f(x)+3f\left(\tfrac{1}{x}\right)=x^2+1, \qquad x\neq 0,$$

and $y=5x^2f(x)$.

Replace x by $1/x$:

$$2f\left(\tfrac{1}{x}\right)+3f(x)=\tfrac{1}{x^2}+1.$$

Solve these two linear equations for $f(x)$:

$$f(x)=\frac{-2x^2+3/x^2+1}{5}.$$

Hence

$$y=5x^2f(x)=x^2\left(-2x^2+\frac{3}{x^2}+1\right)=-2x^4+x^2+3.$$

Differentiate:

$$y'=-8x^3+2x=2x(1-4x^2).$$

Thus $y'>0$ for $x\in(-\infty,-\frac{1}{2})\cup(0,\frac{1}{2})$ (note $x\neq 0$).
Among the options, the strictly increasing interval is

$$\boxed{(0,\tfrac{1}{2})}.$$

Question49

The approximate value of $\cos(59^\circ 30')$ is (given $1^\circ = 0.0175^c, \sin 60^\circ = 0.8660$) MHT CET 2025 (20 Apr Shift 2)

Options:

A. 0.5076

B. 0.5176

C. 0.5256

D. 0.5150

Answer: A

Solution:

$59^\circ 30' = 60^\circ - 0.5^\circ$.

Let $\delta = 0.5^\circ$. Using $1^\circ \approx 0.0175$ rad, $\delta \approx 0.00875$ rad.

$$\cos(60^\circ - \delta) = \cos 60^\circ \cos\delta + \sin 60^\circ \sin\delta.$$

With $\cos 60^\circ = \frac{1}{2}$, $\sin 60^\circ = 0.8660$, and small-angle $\cos\delta \approx 1 - \delta^2/2$, $\sin\delta \approx \delta$:

$$\cos\delta \approx 1 - \frac{(0.00875)^2}{2} \approx 0.999962, \quad \sin\delta \approx 0.00875.$$

So

$$\cos(59^\circ 30') \approx \frac{1}{2}(0.999962) + 0.8660(0.00875) \approx 0.5076.$$

Answer: 0.5076.

Question50

If the curve $y = ax^2 - 6x + b$ passes through $(0, 4)$ and has its tangent parallel to the x -axis at $x = \frac{3}{2}$, then the value of a and b respectively are MHT CET 2025 (20 Apr Shift 2)

Options:

A. $-2, -4$

B. $2, 4$

C. $-2, 4$

D. $2, -4$

Answer: B

Solution:

Slope zero at $x = \frac{3}{2}$:

$$y' = 2ax - 6, \quad 2a \cdot \frac{3}{2} - 6 = 0 \Rightarrow 3a - 6 = 0 \Rightarrow a = 2.$$

Passing through $(0, 4)$ gives $b = 4$.

Answer: $(a, b) = (2, 4)$.

Question51

20 is divided into two parts so that the product of the cube of one part and the square of the other part is maximum, then these two parts are MHT CET 2025 (20 Apr Shift 1)

Options:

A. 15,5

B. 16,4

C. 12,8

D. 14,6

Answer: C

Solution:

Let the parts be x and $20 - x$.

Maximize $P = x^3(20 - x)^2$ (the other case is symmetric).

$$\frac{dP}{dx} = 5x^2(x - 20)(x - 12) = 0 \Rightarrow x = 12 \text{ (interior)}.$$

Second derivative at $x = 12$ is negative ⇒ maximum.

Thus the parts are 12 and 8. (Option C)

Question52

The shortest distance between the line $y - x = 1$ and the curve $x = y^2$ is MHT CET 2025 (20 Apr Shift 1)

Options:

A. $\frac{3\sqrt{2}}{8}$

B. $\frac{2\sqrt{3}}{8}$

C. $\frac{3\sqrt{2}}{5}$

D. $\frac{\sqrt{3}}{4}$

Answer: A

Solution:

$$\text{Line: } y - x = 1 \Rightarrow y = x + 1, \qquad \text{Parabola: } x = y^2$$

Distance from a point (y^2, y) on the parabola to the line is the perpendicular distance:

$$d(y) = \frac{|y - x - 1|}{\sqrt{1^2 + (-1)^2}} = \frac{|y - y^2 - 1|}{\sqrt{2}}.$$

Since $y - y^2 - 1 = -(y^2 - y + 1) = -((y - \frac{1}{2})^2 + \frac{3}{4})$ is always negative,

$$d(y) = \frac{y^2 - y + 1}{\sqrt{2}}.$$

Minimize $g(y) = y^2 - y + 1$: $g'(y) = 2y - 1 = 0 \Rightarrow y = \frac{1}{2}$.

$$g_{\min} = \left(\tfrac{1}{2}\right)^2 - \tfrac{1}{2} + 1 = \tfrac{3}{4}.$$

Thus

$$d_{\min} = \frac{3/4}{\sqrt{2}} = \frac{3\sqrt{2}}{8}.$$

$\boxed{\frac{3\sqrt{2}}{8}}$.

Question53

If the curves $y^2 = 6x$ and $9x^2 +$ by $^2 = 16$ intersect each other at right angles, then the value of b is MHT CET 2025 (20 Apr Shift 1)

Options:

A. 4

B. $\frac{7}{2}$

C. 6

D. $\frac{9}{2}$

Answer: D

Solution:

Let the curves be

$$C_1 : y^2 = 6x \quad \Rightarrow \quad y' = \frac{3}{y},$$

$$C_2 : 9x^2 + by^2 = 16 \quad \Rightarrow \quad 18x + 2byy' = 0 \Rightarrow y' = -\frac{9x}{by}.$$

For orthogonal intersection: $m_1 m_2 = -1$:

$$\frac{3}{y} \cdot \left(-\frac{9x}{by}\right) = -1 \Rightarrow \frac{27x}{by^2} = 1 \Rightarrow x = \frac{by^2}{27}.$$

But from C_1, $x = \frac{y^2}{6}$. Equate:

$$\frac{y^2}{6} = \frac{by^2}{27} \Rightarrow b = \frac{27}{6} = \boxed{\frac{9}{2}}.$$

Question54

The position of a point in time t is given by $x = \mathrm{a} + \mathrm{bt} - \mathrm{ct}^2, \mathrm{y} = \mathrm{at} + \mathrm{bt}^2$.It's resultant acceleration at time t in seconds is given by MHT CET 2025 (19 Apr Shift 2)

Options:

A. $\mathrm{b} - \mathrm{c} \quad \text{unit/seconds}^2$

B. $\mathrm{b} + \mathrm{c} \quad \text{unit/seconds}^2$

C. $2\mathrm{~b} - 2\mathrm{c} \quad \text{unit / seconds}^2$

D. $2\sqrt{b^2 + c^2}$ unit/seconds ^{2}

Answer: D

Solution:

Acceleration components are the second derivatives:

- $x(t) = a + bt - ct^2 \Rightarrow x''(t) = -2c$
- $y(t) = at + bt^2 \Rightarrow y''(t) = 2b$

Resultant acceleration magnitude:

$$\sqrt{(x'')^2 + (y'')^2} = \sqrt{(-2c)^2 + (2b)^2} = 2\sqrt{b^2 + c^2}.$$

Answer: $2\sqrt{b^2 + c^2}\ \mathrm{unit/s}^2$ (Option D).

Question55

A normal is drawn at a point $\mathrm{P}(x, y)$ of a curve $\mathrm{y} = \mathrm{f}(x)$. The normal meets the X axis at $\mathrm{Q}. l(\mathrm{PQ}) = \mathrm{k}$. ($\mathrm{k}$ is a constant)Then equation of the curve through $(0, \mathrm{k})$ is MHT CET 2025 (19 Apr Shift 2)

Options:

A. $x^2 + \mathrm{y}^2 = \mathrm{k}^2$

B. $(1+\mathrm{k})x^2+\mathrm{y}^2=\mathrm{k}^2$

C. $x^2+\left(1+\mathrm{k}^2\right)\mathrm{y}^2=\mathrm{k}^2$

D. $x^2+2\mathrm{y}^2=2\mathrm{k}^2$

Answer: A

Solution:

Let $P(x,y)$ be on $y=f(x)$.
Slope of the tangent is $y'=\frac{dy}{dx}$, so slope of the normal is $-1/y'$.

The normal through (x,y) meets the x-axis at Q. The distance

$$PQ=\sqrt{\left(\frac{y}{-1/y'}\right)^2+y^2}=|y|\sqrt{1+(y')^2}.$$

Given $PQ=k$ (constant),

$$|y|\sqrt{1+(y')^2}=k\ \Rightarrow\ y^2\left(1+(y')^2\right)=k^2.$$

Solve:

$$(y')^2=\frac{k^2-y^2}{y^2}\quad\Rightarrow\quad\frac{dx}{dy}=\frac{y}{\sqrt{k^2-y^2}}.$$

Integrate:

$$x=\int\frac{y}{\sqrt{k^2-y^2}}\,dy=-\sqrt{k^2-y^2}+C.$$

Through $(0,k)$ gives $C=0$, hence

$$x^2+y^2=k^2.$$

$\boxed{x^2+y^2=k^2}$.

Question56

The equation of tangent to the curve $\mathrm{y}=\cos(x+\mathrm{y})$ where $-2\pi\le x\le 2\pi$ and which is parallel to the line $x+2\mathrm{y}=0$, is MHT CET 2025 (19 Apr Shift 2)

Options:

A. $2x+4y+\pi=0$

B. $2x+4y-\pi=0$

C. $2x+4y-3\pi=0$

D. $2x-4y+3\pi=0$

Answer: B

Solution:

Given $y = \cos(x+y)$.
Differentiate implicitly:

$$\frac{dy}{dx} = -\sin(x+y)(1+\frac{dy}{dx}) \Rightarrow \frac{dy}{dx} = \frac{-\sin(x+y)}{1+\sin(x+y)}.$$

For the tangent to be parallel to $x + 2y = 0$ (slope $-\frac{1}{2}$):

$$\frac{-\sin(x+y)}{1+\sin(x+y)} = -\frac{1}{2} \Rightarrow \sin(x+y) = 1.$$

So $x + y = \frac{\pi}{2} + 2k\pi$ and then

$$y = \cos(x+y) = \cos\left(\frac{\pi}{2} + 2k\pi\right) = 0.$$

With $-2\pi \le x \le 2\pi$, $x = \frac{\pi}{2}$ or $-\frac{3\pi}{2}$.
The tangent line (slope $-\frac{1}{2}$) through $(\frac{\pi}{2}, 0)$ is

$$y = -\frac{1}{2}\left(x - \frac{\pi}{2}\right) \iff 2x + 4y - \pi = 0.$$

$\boxed{2x + 4y - \pi = 0}$ (option B).

Question57

If the tangent at $(1,7)$ to the curve $x^2 = y - 6$ touches the circle $x^2 + \mathrm{y}^2 + 16\mathrm{x} + 12\mathrm{y} + \mathrm{C} = 0$, then $\mathrm{C} =$ MHT CET 2025 (19 Apr Shift 2)

Options:

A. 85

B. 95

C. 185

D. 195

Answer: B

Solution:

Slope of the parabola $x^2 = y - 6$ is $\frac{dy}{dx} = 2x$.
At $(1,7)$: slope $= 2$. Tangent line: $y - 7 = 2(x-1) \Rightarrow y = 2x + 5$ (i.e., $2x - y + 5 = 0$).

Circle: $x^2 + y^2 + 16x + 12y + C = 0 \Rightarrow (x+8)^2 + (y+6)^2 = 100 - C$.
Center $(-8, -6)$, radius $\sqrt{100 - C}$.

Distance from center to the line $2x - y + 5 = 0$:

$$\frac{|2(-8) - (-6) + 5|}{\sqrt{2^2 + (-1)^2}} = \frac{|-16 + 6 + 5|}{\sqrt{5}} = \sqrt{5}.$$

For tangency, this distance equals the radius:

$$\sqrt{100 - C} = \sqrt{5} \Rightarrow 100 - C = 5 \Rightarrow C = 95.$$

Answer: $C = 95$.

Question58

If two curves $x^2 - 4y^2 = 2$ and $8x^2 = 40 - \mathrm{my}^2$ are orthogonal to each other then $\mathrm{m} =$ MHT CET 2025 (19 Apr Shift 2)

Options:

A. 2

B. 16

C. $\frac{1}{\sqrt{2}}$

D. 4

Answer: B

Solution:

$m = 16$

For the curves

$$C_1: x^2 - 4y^2 = 2 \quad \Rightarrow \quad y' = \frac{x}{4y},$$

$$C_2: 8x^2 + my^2 = 40 \quad \Rightarrow \quad y' = -\frac{8x}{my}.$$

Orthogonal intersection $\Rightarrow m_1 m_2 = -1$:

$$\frac{x}{4y} \cdot \left(-\frac{8x}{my}\right) = -1 \Rightarrow \frac{2x^2}{my^2} = 1 \Rightarrow m = \frac{2x^2}{y^2}. \quad (1)$$

Let $r = \frac{x^2}{y^2}$. From C_1: $x^2 - 4y^2 = 2 \Rightarrow y^2 = \frac{2}{r-4}$.
Plug into C_2:

$$8x^2 + my^2 = 40 \Rightarrow y^2(8r + m) = 40 \Rightarrow \frac{2}{r-4}(8r + m) = 40 \Rightarrow m = 12r - 80. \quad (2)$$

From (1): $m = 2r$. Equate with (2):

$$2r = 12r - 80 \Rightarrow r = 8 \Rightarrow m = 2r = 16.$$

Question59

A population $p(t)$ of 1000 bacteria introduced into a nutrient medium grows according to the relation $p(t) = 1000 + \frac{1000t}{100+t^2}$. The maximum size of this bacterial population is MHT CET 2025 (19 Apr Shift 1)

Options:

A. 1100

B. 1250

C. 1050

D. 950

Answer: C

Solution:

$p(t) = 1000 + \frac{1000t}{100+t^2}$.

Maximize $f(t) = \frac{t}{100+t^2}$:

$$f'(t) = \frac{100-t^2}{(100+t^2)^2} = 0 \Rightarrow t^2 = 100 \Rightarrow t = 10 \text{ (time} \geq 0).$$

$f_{\max} = \frac{10}{200} = \frac{1}{20}$.

Hence $p_{\max} = 1000 + 1000 \cdot \frac{1}{20} = 1050$.

Question60

By dropping a stone in a quiet lake, a wave in the form of circle is generated. The radius of the circular wave increases at the rate of 2.1 cm/sec. Then the rate of increase of the enclosed circular region, when the radius of the circular wave is 10 cm, is (Given $\pi = \frac{22}{7}$) MHT CET 2025 (19 Apr Shift 1)

Options:

A. 66 cm^2/ second

B. 122 cm^2/ second

C. 132 cm^2/ second

D. 110 cm^2/ second

Answer: C

Solution:

Area $A = \pi r^2$.
$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2 \cdot \frac{22}{7} \cdot 10 \cdot 2.1 = 132 \text{ cm}^2/\text{s}$.

Answer: 132 cm^2/s (Option C).

Question61

The angle between the curves $xy = 6$ and $x^2 y = 12$ is MHT CET 2025 (19 Apr Shift 1)

Options:

A. $\tan^{-1} \frac{3}{11}$

B. $\tan^{-1} \frac{11}{3}$

C. $\tan^{-1} \frac{2}{11}$

D. $\tan^{-1} \frac{1}{11}$

Answer: A

Solution:

$$xy = 6 \Rightarrow y = \frac{6}{x}, \quad x^2y = 12$$

Intersection: $x^2 \cdot \frac{6}{x} = 12 \Rightarrow x = 2,\ y = 3.$

Slopes:

- For $xy = 6$: $y + xy' = 0 \Rightarrow y' = -\frac{y}{x} = -\frac{3}{2} = m_1.$
- For $x^2y = 12$: $2xy + x^2y' = 0 \Rightarrow y' = -\frac{2y}{x} = -3 = m_2.$

Angle between tangents:

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \frac{\left|-3 + \frac{3}{2}\right|}{1 + \left(-\frac{3}{2}\right)(-3)} = \frac{\frac{3}{2}}{\frac{11}{2}} = \frac{3}{11}.$$

$$\boxed{\theta = \tan^{-1}\left(\frac{3}{11}\right)}$$

Question62

In the mean value theorem, $f'(c) = \frac{f(b)-f(a)}{b-a}$, if $a = 0,\ b = \frac{1}{2}$ and $f(x) = x(x-1)(x-2)$, then the value of c is MHT CET 2025 (19 Apr Shift 1)

Options:

A. $1 - \frac{\sqrt{15}}{6}$

B. $1 - \frac{\sqrt{13}}{6}$

C. $1 - \frac{\sqrt{21}}{6}$

D. $1 + \frac{\sqrt{21}}{6}$

Answer: C

Solution:

$f(x) = x(x-1)(x-2) = x^3 - 3x^2 + 2x \Rightarrow f'(x) = 3x^2 - 6x + 2.$

On $[0, \frac{1}{2}]$,

$$\frac{f(b) - f(a)}{b - a} = \frac{f(\frac{1}{2}) - f(0)}{\frac{1}{2}} = \frac{\frac{1}{8} - \frac{3}{4} + 1}{\frac{1}{2}} = \frac{\frac{3}{8}}{\frac{1}{2}} = \frac{3}{4}.$$

So $f'(c) = \frac{3}{4}$:

$$3c^2 - 6c + 2 = \frac{3}{4} \Rightarrow 12c^2 - 24c + 5 = 0 \Rightarrow c = \frac{24 \pm 4\sqrt{21}}{24} = 1 \pm \frac{\sqrt{21}}{6}.$$

Only $c \in (0, \frac{1}{2})$ is valid:

$$\boxed{c = 1 - \frac{\sqrt{21}}{6}}.$$

Question63

The rate of change of the volume of a sphere with respect to its surface area, when its radius is 2 cm, is _______ cm^3/cm^2. MHT CET 2024 (16 May Shift 2)

Options:

A. 0.1

B. 0.5

C. 1

D. 2

Answer: C

Solution:

Volume of sphere $(V)=\frac{4}{3}\pi r^3$ Surface area of sphere $(A)=4\pi r^2$

$$\therefore \quad \frac{dV}{dr}=4\pi r^2 \text{ and } \frac{dA}{dr}=8\pi r$$

$$\therefore \quad \frac{dV}{dA}=\frac{\frac{dV}{dr}\frac{dr}{dA}}{\frac{dr}{dr}}=\frac{4\pi r^2}{8\pi r}=\frac{r}{2}$$

$$\therefore \quad \left(\frac{dV}{dA}\right)_{r=2}=\frac{2}{2}=1\ \mathrm{cm}^3/\mathrm{cm}^2$$

Question64

Water is being poured at the rate of $36\ \mathrm{m}^3/\mathrm{min}$ into a cylindrical vessel, whose circular base is of radius 3 meters. Then the water level in the cylinder is rising at the rate of MHT CET 2024 (16 May Shift 2)

Options:

A. $4\pi\ \mathrm{m}/\mathrm{min}$

B. $\frac{4}{\pi}\ \mathrm{m}/\mathrm{min}$

C. $\frac{1}{4\pi}\ \mathrm{m}/\mathrm{min}$

D. $\frac{\pi}{4}\ \mathrm{m}/\mathrm{min}$

Answer: B

Solution:

$$\frac{\mathrm{dV}}{\mathrm{dt}}=36\ \mathrm{m}^3/\mathrm{min},\ \text{radius (r)}=3\ \mathrm{m}$$

$$\text{Volume (V)}=\pi \mathrm{r}^2\ \mathrm{h}$$

$$\therefore \quad \frac{\mathrm{dV}}{\mathrm{dt}}=\pi \mathrm{r}^2\frac{\mathrm{dh}}{\mathrm{dt}}$$

$$\Rightarrow 36=\pi\times(3)^2\frac{\mathrm{dh}}{\mathrm{dt}}$$

$$\Rightarrow \frac{\mathrm{dh}}{\mathrm{dt}}=\frac{4}{\pi}\ \mathrm{m}/\mathrm{min}$$

Question65

If $\mathrm{f}(x)=\frac{\mathrm{a}\sin x+b\cos x}{\mathrm{c}\sin x+\mathrm{d}\cos x}$ is decreasing for all x, then MHT CET 2024 (16 May Shift 2)

Options:

A. $\mathrm{ad}-\mathrm{bc}>0$

B. $\mathrm{ad}-\mathrm{bc}<0$

C. $\mathrm{ab}-\mathrm{cd}>0$

D. $\mathrm{ab}-\mathrm{cd}<0$

Answer: B

Solution:

$$\mathrm{f}(x)=\frac{\mathrm{a}\sin x+\mathrm{b}\cos x}{\mathrm{c}\sin x+\mathrm{d}\cos x}$$

$\mathrm{f}(x)$ will be decreasing, if $\mathrm{f}'(x)<0$

$$\therefore \frac{1}{(\mathrm{c}\sin x+\mathrm{d}\cos x)^2}[(\mathrm{c}\sin x+\mathrm{d}\cos x)(\mathrm{a}\cos x-b\sin x)$$

$$-(a\sin x+b\cos x)(\cos x-d\sin x)]<0$$

$$\Rightarrow \mathrm{ac}\sin x\cos x-\mathrm{bc}\sin^2 x+\mathrm{ad}\cos^2 x$$
$$-\mathrm{bd}\sin x\cos x-\mathrm{ac}\sin x\cos x+\mathrm{ad}\sin^2 x$$

$$-\mathrm{bc}\cos^2 x+\mathrm{bd}\sin x\cos x<0$$

$$\Rightarrow \mathrm{ad}\left(\sin^2 x+\cos^2 x\right)-\mathrm{bc}\left(\sin^2 x+\cos^2 x\right)<0$$
$$\Rightarrow \mathrm{ad}-\mathrm{bc}<0$$

Question66

The equation of the normal to the curve $y=x\log x$ parallel to $2x-2y+3=0$ is MHT CET 2024 (16 May Shift 2)

Options:

A. $x+y=3\mathrm{e}^{-2}$

B. $x-y=3\mathrm{e}^{-2}$

C. $x-y=3\mathrm{e}^{2}$

D. $x+y=3\mathrm{e}^{2}$

Answer: B

Solution:

$$y = x\log x\ldots(i)$$
$$\therefore \quad \frac{dy}{dx} = 1 + \log x$$
Slope of the normal $= \frac{-1}{\frac{dy}{dx}} = -\frac{1}{1+\log x}$

Slope of the given line is 1.

Since the normal is parallel to the given line.

$$\therefore \quad -\frac{1}{1+\log x} = 1$$
$$\Rightarrow \log x = -2$$
$$\Rightarrow x = e^{-2}$$
$$y = -2e^{-2}$$

...[From (i)]

$\therefore$ Equation of the normal at $\left(e^{-2}, -2e^{-2}\right)$ is

$$y + 2e^{-2} = 1\left(x - e^{-2}\right)$$
$$\Rightarrow y + 2e^{-2} = x - e^{-2}$$
$$\Rightarrow x - y = 3e^{-2}$$

Question67

If $x = -1$ and $\cdot x = 2$ are extreme points of $f(x) = \alpha\log|x| + \beta x^2 + x$, then MHT CET 2024 (16 May Shift 2)

Options:

A. $\alpha = -6, \beta = \frac{1}{2}$

B. $\alpha = -6, \beta = -\frac{1}{2}$

C. $\alpha = 2, \beta = -\frac{1}{2}$

D. $\alpha = 2, \beta = \frac{1}{2}$

Answer: C

Solution:

According to the given condition, $f'(-1) = 0$ and $f'(2) = 0$ $f(x) = \alpha\log|x| + \beta x^2 + x$

$$\therefore \quad f'(x) = \frac{\alpha}{x} + 2\beta x + 1$$
$$\therefore \quad f'(-1) = 0 \Rightarrow \alpha + 2\beta = 1\ldots(i)$$
$$\text{and } f'(2) = 0 \Rightarrow \alpha + 8\beta = -2\ldots(ii)$$
From (i) and (ii), we get

$$\alpha = 2 \text{ and } \beta = \frac{-1}{2}$$

Question68

If θ denotes the acute angle between the curves $y = 10 - x^2$ and $y = 2 + x^2$, at a point of the intersection, then $|\tan\theta|$ is equal to MHT CET 2024 (16 May Shift 1)

Options:

A. $\frac{8}{15}$

B. $\frac{8}{17}$

C. $\frac{4}{9}$

D. $\frac{7}{17}$

Answer: A

Solution:

$y = 10 - x^2 \ldots (i)$
$y = 2 + x^2 \ldots (ii)$
Solving (i) and (ii), we get $y = 6$

From (i), $6 = 10 - x^2 \Rightarrow x = \pm 2$ Differentiating (i) w.r.t. x, we get

$\frac{dy}{dx} = -2x \Rightarrow \left(\frac{dy}{dx}\right)_{(-2,6)} = 4$
Differentiating (ii) w.r.t. x, we get

$$\frac{dy}{dx} = 2x \Rightarrow \left(\frac{dy}{dx}\right)_{(-2,6)} = -4$$
$$\therefore \quad \tan\theta = \left|\frac{4-(-4)}{1+4(-4)}\right| = \left|\frac{8}{-15}\right| \Rightarrow |\tan\theta| = \frac{8}{15}$$

Question69

If $y = \mathrm{a}\log x + \mathrm{b}x^2 + x$ has its extremum values at $x = -1$ and $x = 2$, then MHT CET 2024 (16 May Shift 1)

Options:

A. $\mathrm{a} = 2,\ \mathrm{b} = -1$

B. $\mathrm{a} = 2,\ \mathrm{b} = -\frac{1}{2}$

C. $\mathrm{a} = -2,\ \mathrm{b} = \frac{1}{2}$

D. $\mathrm{a} = 2,\ \mathrm{b} = \frac{1}{2}$

Answer: B

Solution:

$$y=a\log x+bx^2+x$$

$\therefore \quad \frac{dy}{dx}=\frac{a}{x}+2bx+1$

$\Rightarrow\left(\frac{dy}{dx}\right)_{x=-1}=-a-2b+1=0$

$\Rightarrow a+2b=1\ldots(i)$

and $\left(\frac{dy}{dx}\right)_{x=2}=\frac{a}{2}+4b+1=0$

$\Rightarrow a+8b+2=0\ldots(ii)$

Solving (i) and (ii), we get

$a=2, b=-\frac{1}{2}$

Question70

The set of all points, for which $f(x)=x^2e^{-\dot{x}}$ strictly increases, is MHT CET 2024 (16 May Shift 1)

Options:

A. $(0,2)$

B. $(2,\infty)$

C. $(-2,0)$

D. $(-\infty,\infty)$

Answer: A

Solution:

$f'(x)=2xe^{-x}-x^2e^{-x}=xe^{-x}(2-x)$

Since f is increasing, $f'(x)>0$

$\Rightarrow xe^{-x}(2-x)>0$

$\Rightarrow x(2-x)>0$

$\Rightarrow 0<x<2$

$\Rightarrow x\in(0,2)$

Question71

An open tank with a square bottom, to contain 4000 cubic cm . of liquid, is to be constructed. The dimensions of the tank, so that the surface area of the tank is minimum, are MHT CET 2024 (15 May Shift 2)

Options:

A. side of square bottom $=40$ cm, height $=10$ cm.

B. side of square bottom $=20$ cm, height $=10$ cm.

C. side of square bottom $=10$ cm, height $=40$ cm.

D. side. of square bottom $=5$ cm, height $=160$ cm.

Answer: B

Solution:

Let x be the length of the side of square bottom, h be the height, V be the volume and A be the surface area of open tank.

Then,

$$V = x^2 h = 4000 \ldots (i)$$
$$A = x^2 + 4x h \ldots (ii)$$

From (i),

$h = \frac{4000}{x^2}$

Substituting the value of h in (ii), we get

$$A = x^2 + \frac{16000}{x}$$
$$\therefore \frac{dA}{dx} = 2x - \frac{16000}{x^2}$$

A is minimum, if $\frac{dA}{dx} = 0$

$$\Rightarrow 2x - \frac{16000}{x^2} = 0$$
$$\Rightarrow x^3 = 8000$$
$$\Rightarrow x = 20$$

Now, $\frac{d^2 A}{dx^2} = 2 + \frac{32000}{x^3}$

$$\Rightarrow \left(\frac{d^2 A}{dx^2}\right)_{x=20} = 6 > 0$$

$\therefore$ A is minimum when $x = 20$ cm.

$h = \frac{4000}{x^2} = \frac{4000}{400} = 10$ cm

Question72

The function $f(x) = 2x^3 - 9x^2 + 12x + 2$ is decreasing in MHT CET 2024 (15 May Shift 2)

Options:

A. $1 < x < 2$

B. $x < 1$ or $x > 2$

C. $x < -1$ or $x > -2$

D. $-2 < x < -1$

Answer: A

Solution:

$$f(x) = 2x^3 - 9x^2 + 12x + 2$$
$$\therefore f'(x) = 6x^2 - 18x + 12$$

For decreasing function, $f'(x) < 0$

$$\Rightarrow 6x^2 - 18x + 12 < 0$$
$$\Rightarrow x^2 - 3x + 2 < 0$$
$$\Rightarrow (x-1)(x-2) < 0$$
$$\Rightarrow x \in (1, 2)$$

Question73

If $f(x) = x^3 - 10x^2 + 200x - 10$, then MHT CET 2024 (15 May Shift 1)

Options:

A. $f(x)$ is decreasing in $(-\infty, 10]$ and increasing in $[10, \infty)$

B. $f(x)$ is increasing in $(-\infty, 10]$ and decreasing in $[10, \infty)$

C. $f(x)$ is increasing throughout real line

D. $f(x)$ is decreasing throughout real line

Answer: C

Solution:

$$f(x) = x^3 - 10x^2 + 200x - 10$$
$$\Rightarrow f'(x) = 3x^2 - 20x + 200$$

For $f(x)$ to be increasing $f'(x) > 0$

$$\Rightarrow 3x^2 - 20x + 200 > 0$$
$$\Rightarrow 3\left(x^2 - \frac{20}{3}x + \frac{200}{3} + \frac{100}{9} - \frac{100}{9}\right) > 0$$
$$\Rightarrow 3\left[\left(x - \frac{10}{3}\right)^2 + \frac{500}{9}\right] > 0$$
$$\Rightarrow 3\left(x - \frac{10}{3}\right)^2 + \frac{500}{3} > 0$$

Always increasing throughout real line.

Question74

Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq.m) of the flowerbed is MHT CET 2024 (15 May Shift 1)

Options:

A. 30

B. 12.5

C. 25

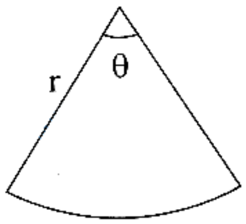
D. 10

Answer: C

Solution:

Total length of wire $= r + r + r\theta$

$$\Rightarrow 20 = 2r + r\theta$$
$$\Rightarrow \theta = \frac{20 - 2r}{r}$$



$$A = \frac{1}{2}r^2\theta$$
$$= \frac{1}{2}r^2\left(\frac{20-2r}{r}\right) = 10r - r^2$$
$$\therefore \quad \frac{dA}{dr} = 10 - 2r$$

For maximum area, $\frac{dA}{dr} = 0$

$$\Rightarrow 10 - 2r = 0$$
$$\Rightarrow r = 5$$
$$\frac{d^2A}{dr^2} = -2 < 0$$

Area is maximum at $r = 5$

$$\therefore \quad \text{Maximum area } = 10(5) - 5^2$$
$$= 50 - 25 = 25 \text{sq. m}$$

Question75

A ladder 5 m in length is leaning against a wall. The bottom of the ladder is pulled along the ground away from the wall, at the rate of 2 m/sec. How fast is the height on the wall decreasing when the foot of the ladder is 4 m away from the wall? MHT CET 2024 (15 May Shift 1)

Options:

A. $\frac{4}{3}$ m/sec

B. $\frac{2}{3}$ m/sec

C. $\frac{5}{3}$ m/sec

D. $\frac{8}{3}$ m/sec

Answer: D

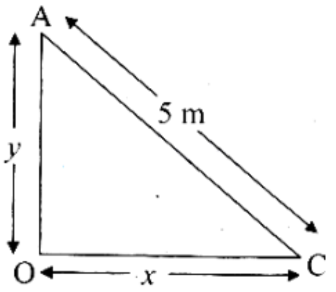
Solution:

In right angled $\triangle AOC$,

$$x^2 + y^2 = (5)^2$$
$$\Rightarrow y^2 = 5^2 - x^2$$

Differentiating w.r.t. t, we get

$$2y\frac{dy}{dt} = -2x\frac{dx}{dt}$$
$$\Rightarrow \frac{dy}{dt} = -\frac{x}{y} \cdot \frac{dx}{dt}$$
$$= \frac{-x}{\sqrt{5^2 - x^2}} \cdot \frac{dx}{dt}$$



$$\therefore \quad \left(\frac{dy}{dt}\right)_{x=4} = \frac{-4}{\sqrt{25-16}} \cdot (2)$$
$$= \frac{-8}{3} \text{ m/sec}$$

Thus; the height on the wall is decreasing at the rate of $\frac{8}{3}$ m/sec

---

Question76

The approximate value of $3^{2.001}$, if $\log 3 = 1.0986$ is MHT CET 2024 (15 May Shift 1)

Options:

A. 9.00898

B. 9.0989

C. 9.0898

D. 9.00989

Answer: D

Solution:

$$\begin{aligned} &\text{Let } f(x) \\ \therefore \quad & = 3^x \\ f'(x) &= 3^x \log 3 \\ \therefore \quad \text{Here, } a &= 2 \text{ and } h = 0.001 \\ \therefore \quad f(a+h) &\approx f(a) + hf'(a) \\ &\approx f(2) + (0.001) + f'(2) \\ &\approx 3^2 + (0.001)\left(3^2 \log 3\right) \\ &\approx 9 + (0.001)(9 \times 1.0986) \\ &\approx 9 + 0.00989 \\ &\approx 9.00989 \end{aligned}$$

Question77

If $y = a \log x + bx^2 + x$ has its extreme values at $x = -1$ and $x = 2$, then the value of $\left(\frac{a}{b} + \frac{b}{a}\right)$ is MHT CET 2024 (11 May Shift 2)

Options:

A. $-\frac{7}{4}$

B. $-\frac{15}{4}$

C. $-\frac{17}{4}$

D. $-\frac{5}{4}$

Answer: C

Solution:

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{a}}{x}+2\,\mathrm{b}x+1 \Rightarrow\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{x=-1}=-\mathrm{a}-2\,\mathrm{b}+1=0$$

$$\Rightarrow \mathrm{a}=-2\,\mathrm{b}+1$$

$$\text{and}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{x=2}=\frac{\mathrm{a}}{2}+4\,\mathrm{b}+1=0$$

$$\Rightarrow \frac{-2\,\mathrm{b}+1}{2}+4\,\mathrm{b}+1=0$$

$$\Rightarrow -\mathrm{b}+4\,\mathrm{b}+\frac{3}{2}=0$$

$$\Rightarrow 3\,\mathrm{b}=\frac{-3}{2} \Rightarrow\ \mathrm{b}=\frac{-1}{2} \text{ and } \mathrm{a}=2$$

$$\Rightarrow\left(\frac{\mathrm{a}}{\mathrm{b}}+\frac{\mathrm{b}}{\mathrm{a}}\right)=\frac{-17}{4}$$

Question78

The curve $y=ax^3+bx^2+cx+5$ touches the X-axis at $(-2,0)$ and cuts the Y'-axis at a point Q where its gradient is 3 , then values of $\mathrm{a}, \mathrm{b}, \mathrm{c}$ respectively, are MHT CET 2024 (11 May Shift 2)

Options:

A. $3,-\frac{1}{2},-\frac{3}{4}$

B. $-\frac{3}{4},-\frac{1}{2}, 3$

C. $-\frac{1}{2},-\frac{3}{4}, 3$

D. $-\frac{1}{2}, 3,-\frac{3}{4}$

Answer: C

Solution:

$y=ax^3+bx^2+cx+5$ touches X-axis at $(-2,0)$
$\Rightarrow 0=-8a+4b-2c+5$
$\Rightarrow 8\mathrm{a}-4b+2c=5\ldots(i)$
Also, it cuts Y-axis at a point Q

$\therefore$ Put $x=0$ in the equation of curve, we get $y=5$

$$
\begin{aligned}
\therefore \quad & \mathrm{Q} \equiv(0,5) \\
& y=\mathrm{a}x^3+\mathrm{b}x^2+\mathrm{c}x+5 \\
\therefore \quad & \frac{\mathrm{d}y}{\mathrm{d}x}=3\mathrm{a}x^2+2\,\mathrm{b}x+\mathrm{c} \\
& \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{\mathrm{Q}(0,5)}=3 \\
& \Rightarrow 3\mathrm{a}(0)^2+2\,\mathrm{b}(0)+\mathrm{c}=3 \\
& \Rightarrow \mathrm{c}=3
\end{aligned}
$$

Equation (i) becomes,

$8a-4b+6=5$
$\Rightarrow 8a-4b+1=0\ldots(ii)$
Option (C) satisfies equation (ii)

$\therefore$ Option (C) is correct

Question79

The maximum value of the function $\mathrm{f}(x)=2x^3-15x^2+36x-48$ on the set $\mathrm{A}=\left\{x \mid x^2+20 \leq 9x\right\}$ is MHT CET 2024 (11 May Shift 2)

Options:

A. -16

B. -7

C. 16

D. 7

Answer: D

Solution:

$$
\begin{aligned}
& \text{Let } \mathrm{f}(x)=2x^3-15x^2+36x-48 \\
\therefore \quad & \mathrm{f}'(x)=6x^2-30x+36=0 \text{ at } x=3,2 \\
\therefore \quad & \mathrm{f}''(x)=12x-30 \\
& \mathrm{A}=\left\{x \mid x^2-9x+20 \leq 0\right\}=[4,5] \\
\therefore \quad & 2,3 \notin \mathrm{A} \\
\therefore \quad & \text{At } x=4, \mathrm{f}(x)=-16 \text{ and at } x=5, \mathrm{f}(x)=7 \\
\therefore \quad & \text{Maximum value of } \mathrm{f}(x) \text{ is at } x=5 \\
\therefore \quad & \text{Maximum value of } \mathrm{f}(x) \text{ is } 7.
\end{aligned}
$$

Question80

If the normal to the curve $y=\mathrm{f}(x)$ at the point (3,4) makes an angle of $\left(\frac{3\pi}{4}\right)$ with the positive X - axis, then the value of $\mathrm{f}'(3)$ is MHT CET 2024 (11 May Shift 2)

Options:

A. -1

B. $-\frac{3}{4}$

C. $\frac{4}{3}$

D. 1

Answer: D

Solution:

$$\text{Slope of the normal } = \frac{-1}{\frac{\mathrm{d}y}{\mathrm{d}x}}$$

$$\Rightarrow \tan\frac{3\pi}{4} = \frac{-1}{\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{(3,4)}}$$

$$\Rightarrow \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{(3,4)} = \mathrm{i}$$

$$\Rightarrow \mathrm{f}'(3) = 1$$

Question81

Let $\mathrm{f}(x)=\frac{x}{\sqrt{\mathrm{a}^2+x^2}}-\frac{\mathrm{d}-x}{\sqrt{\mathrm{b}^2+(\mathrm{d}-x)^2}}, x\in\mathbb{R}$ where a, b, d are non-zero real constants. Then MHT CET 2024 (11 May Shift 2)

Options:

A. f' is not a continuous function of x.

B. f is neither increasing nor decreasing function of x.

C. f is an increasing function of x.

D. f is a decreasing function of x.

Answer: C

Solution:

$$f(x)=\frac{x}{\sqrt{a^2+x^2}}-\frac{d-x}{\sqrt{b^2+(d-x)^2}}$$

$$\therefore \quad f'(x)=\frac{\sqrt{a^2+x^2}-\frac{x\times 2x}{2\sqrt{a^2+x^2}}}{a^2+x^2}$$

$$-\frac{(-1)\sqrt{b^2+(d-x)^2}+\frac{2(d-x)^2}{2\sqrt{b^2+(d-x)^2}}}{[b^2+(d-x)^2]}$$

$$=\frac{a^2+x^2-x^2}{(a^2+x^2)\sqrt{a^2+x^2}}-\frac{-[b^2+(d-x)^2]+(d-x)^2}{[b^2+(d-x)^2]\sqrt{b^2+(d-x)^2}}$$

$$=\frac{a^2}{(a^2+x^2)^{\frac{3}{2}}}+\frac{b^2}{\left[b^2+(d-x)^2\right]^{\frac{3}{2}}}$$

$$>0 \quad \forall x\in R$$

$\therefore$ $f(x)$ is an increasing function of x.

Question82

If $f(x)=x^3-6x^2+9x+3$ is monotonically decreasing function, then x lies in MHT CET 2024 (11 May Shift 1)

Options:

A. $(3,\infty)$

B. $(1,3)$

C. $[3,\infty)$

D. $[0,3]$

Answer: B

Solution:

$$f(x)=x^3-6x^2+9x+3$$

$$\therefore \quad f'(x)=3x^2-12x+9$$

Since $f(x)$ is monotonically decreasing,

$$f'(x)<0$$
$$\Rightarrow 3x^2-12x+9<0$$
$$\Rightarrow x^2-4x+3<0$$
$$\Rightarrow (x-3)(x-1)<0$$
$$\Rightarrow x\in(1,3)$$

Question83

If equation of normal to the curve $x=\sqrt{t}$, $y=t-\frac{1}{\sqrt{t}}$ at $t=4$ is MHT CET 2024 (11 May Shift 1)

Options:

A. $8x+2y=23$

B. $34x - 8y = 40$

C. $8x + 6y = 37$

D. $8x + 34y = 135$

Answer: D

Solution:

$x = \sqrt{t}, y = t - \frac{1}{\sqrt{t}}$

$\therefore$ $y = x^2 - \frac{1}{x}$, at $t = 4, x = 2$ and $y = \frac{7}{2}$

$\therefore$ $\frac{dy}{dx} = 2x + \frac{1}{x^2}$

$\therefore$ Slope of the normal at $t = 4$ is $\frac{-1}{\left(\frac{dy}{dx}\right)_{t=4}} = \frac{-4}{17}$

$\therefore$ Required equation is $\left(y - \frac{7}{2}\right) = \frac{-4}{17}(x - 2)$ i.e., $8x + 34y = 135$

Question84

The approximate value of $(3.978)^{\frac{3}{2}}$ is MHT CET 2024 (11 May Shift 1)

Options:

A. 7.934

B. 8.934

C. 7.022

D. 8.866°

Answer: A

Solution:

$$\text{Let } f(x) = x^{\frac{3}{2}}$$
$$\therefore \quad f'(x) = \frac{3}{2}x^{\frac{1}{2}}$$
$$\text{Here, } a = 4, \; h = -0.022$$
$$f(3.978) = (4)^{\frac{3}{2}} = -0.022 \times \frac{3}{2} \times (4)^{\frac{1}{2}}$$
$$= 8 - 0.033 \times 2$$
$$= 8 - 0.066 = 7.934$$

Question85

The rate of change of the volume of a sphere with respect to its surface area, when its radius is 2 cm , is MHT CET 2024 (11 May Shift 1)

Options:

A. $0.1 \mathrm{~cm}^3 / \mathrm{cm}^2$

B. $\frac{1}{2} \mathrm{~cm}^3 / \mathrm{cm}^2$

C. $1 \mathrm{~cm}^3 / \mathrm{cm}^2$

D. $2 \mathrm{~cm}^3 / \mathrm{cm}^2$

Answer: C

Solution:

Volume of sphere $(V)=\frac{4}{3} \pi r^3$

Surface area of sphere $(A)=4 \pi r^2$

$\frac{\mathrm{dV}}{\mathrm{dr}}=4 \pi \mathrm{r}^2$ and $\frac{\mathrm{dA}}{\mathrm{dr}}=8 \pi \mathrm{r}$

$$\therefore \quad\left(\frac{d V}{d A}\right)=\frac{\left(\frac{d V}{d r}\right)}{\left(\frac{d A}{d r}\right)}=\frac{4 \pi r^2}{8 \pi r}=\frac{r}{2}$$

$$\therefore \quad\left(\frac{d V}{d A}\right)_{r=2}=1 \mathrm{~cm}^3 / \mathrm{cm}^2$$

Question86

The distance ' s ' in meters covered by a body in t seconds is given by $s=3 t^2-8 t+5$. The body will stop after MHT CET 2024 (11 May Shift 1)

Options:

A. 1 sec

B. $\frac{3}{4}$sec

C. $\frac{4}{3}$sec

D. 4 sec

Answer: C

Solution:

$$s=3 t^2-8 t+5$$

$$\therefore \quad \frac{d s}{d t}=6 t-8$$

when body stops, $\frac{\mathrm{ds}}{\mathrm{dt}}=0$

$\therefore \quad 6 \mathrm{t}-8=0 \Rightarrow t=\frac{4}{3}$

Question87

The volume of a ball is increasing at the rate of $4 \pi \mathrm{cc} / \mathrm{sec}$. The rate of increase of the radius, when the volume is $288 \pi \mathrm{cc}$, is MHT CET 2024 (10 May Shift 2)

Options:

A. $\frac{1}{6}$ cm/sec

B. $\frac{1}{36}$ cm/sec

C. 6 cm/sec

D. 36 cm/sec

Answer: B

Solution:

$$V=\frac{4}{3}\pi r^3$$
$$\Rightarrow 288\pi=\frac{4}{3}\pi r^3$$
$$\Rightarrow r=6\text{ cm}$$

$$\mathrm{V}=\frac{4}{3}\pi\mathrm{r}^3$$
$$\therefore \frac{\mathrm{dV}}{\mathrm{dt}}=4\pi\mathrm{r}^2\cdot\frac{\mathrm{dr}}{\mathrm{dt}}$$
$$\Rightarrow 4\pi=4\pi\mathrm{r}^2\frac{\mathrm{dr}}{\mathrm{dt}}$$
$$\therefore \frac{\mathrm{dr}}{\mathrm{dt}}=\frac{1}{\mathrm{r}^2}=\frac{1}{36}\text{ cm/sec}$$

Question88

Let $\mathrm{f}(x)=(x-1)(x-2)(x-3), x\in[0,4]$, Values of C will be _______ [if L.M.V.T. (Lagrange's Mean Value Theorem) can be applied]. MHT CET 2024 (10 May Shift 2)

Options:

A. $\frac{4-2\sqrt{3}}{3}, \frac{4+2\sqrt{3}}{3}$

B. $\frac{6-2\sqrt{3}}{3}, \frac{6+2\sqrt{3}}{3}$

C. $\frac{6-\sqrt{3}}{3}, \frac{6+\sqrt{3}}{3}$

D. $2-\sqrt{3}, 2+\sqrt{3}$

Answer: B

Solution:

Let $y=(x-1)(x-2)(x-3)$

$\log y=\log(x-1)+\log(x-2)+\log(x-3)$

Differentiating w.r.t. x, we get $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{x-1}+\frac{1}{x-2}+\frac{1}{x-3}$

$$\therefore \quad f'(x) = \frac{dy}{dx} = (x-2)(x-3) + (x-1)(x-3)$$
$$+ (x-1)(x-2)$$
$$\therefore \quad f'(c) = \frac{f(4)-f(0)}{4-0} = \frac{6-(-6)}{4} = 3, \text{ for } c \in [0,4]$$
$$\therefore \quad (c-2)(c-3) + (c-1)(c-3) + (c-1)(c-2) = 3$$
$$\therefore \quad 3c^2 - 12c + 11 = 3$$
$$\therefore \quad 3c^2 - 12c + 8 = 0$$
$$\therefore \quad c = \frac{12 \pm \sqrt{144-96}}{6} = \frac{6 \pm 2\sqrt{3}}{3}$$

Question89

If $y = 4x - 5$ is a tangent to the curve $y^2 = px^3 + q$ at $(2,3)$, then the values of p and q are respectively MHT CET 2024 (10 May Shift 2)

Options:

A. $-2, 7$

B. $7, -2$

C. $2, -7$

D. $-7, -2$

Answer: C

Solution:

$y^2 = px^3 + q \ldots (i)$

Differentiating both sides w.r.t. x, we get

$$2y \cdot \frac{dy}{dx} = 3px^2$$
$$\Rightarrow \frac{dy}{dx} = \frac{3p}{2}\left(\frac{x^2}{y}\right)$$
$$\therefore \left(\frac{dy}{dx}\right)_{(2,3)} = \frac{3p}{2} \times \frac{4}{3} = 2p$$

Since the line touches the curve, their slopes are equal.

$\therefore \quad 2p = 4 \Rightarrow p = 2$

Since $(2,3)$ lies on $y^2 = px^3 + q$.

$\therefore \quad 9 = 2 \times 8 + q \Rightarrow q = -7$

Question90

The curve $x^4 - 2xy^2 + y^2 + 3x - 3y = 0$ cuts the X -axis at $(0,0)$ at an angle of MHT CET 2024 (10 May Shift 1)

Options:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. 0

D. $\frac{\pi}{6}$

Answer: A

Solution:

Given equation of curve is

$x^4 - 2xy^2 + y^2 + 3x - 3y = 0 \ldots(i)$

Slope of tangent $= m = \frac{dy}{dx}$ at $(0,0)$

Differentiating (i) w.r.to x, we get

$$4x^3 - 2x \cdot 2y\frac{dy}{dx} - y^2(2) + 2y\frac{dy}{dx} + 3 - 3\frac{dy}{dx} = 0$$
$$\therefore \quad \frac{dy}{dx} = \frac{4x^3 - 2y^2 + 3}{4xy - 2y + 3}$$
$$\therefore \quad m = \left.\frac{dy}{dx}\right|_{(0,0)} = \frac{4(0) - 2(0) + 3}{4(0)\cdot(0) - 2(0) + 3} = \frac{3}{3} = 1$$
$$\therefore \quad m = 1$$
$$\therefore \quad \tan\theta = 1$$
$$\therefore \quad \theta = \frac{\pi}{4}$$

Question91

Water is running in a hemispherical bowl of radius 180 cm at the rate of 108 cubic decimeters per minute. How fast the water level is rising when depth of the water level in the bowl is 120 cm ? (1 decimeter $= 10$ cm) MHT CET 2024 (10 May Shift 1)

Options:

A. 16π cm/s

B. $\frac{16}{\pi}$ cm/s

C. $\frac{1}{16\pi}$ cm/s

D. $\frac{\pi}{16}$ cm/s

Answer: C

Solution:

Radius of hemispherical bowl $(\mathrm{r})=180\ \mathrm{cm}$

Rate of flow

$$\begin{aligned}\left(\frac{\mathrm{dV}}{\mathrm{dt}}\right) &= 108\mathrm{dm}^3/\mathrm{min}\\ &= 108000\ \mathrm{cm}^3/\mathrm{min}\\ &= \frac{108000}{60}\ \mathrm{cm}^3/\mathrm{sec}\\ &= 1800\ \mathrm{cm}^3/\mathrm{sec}\end{aligned}$$

Let depth of water in bowl be x.

$\therefore$ Volume of water in hemispherical

$$\text{bowl }(\mathrm{V})=\frac{\pi}{3}x^2(3r-x)$$

$$\mathrm{V}=\frac{\pi}{3}x^2(3\times 180-x)$$

$$\therefore \quad \mathrm{V}=180\pi x^2-\frac{\pi}{3}x^3$$

Differentiating w.r.t. x, we get

$$\frac{\mathrm{dV}}{\mathrm{dt}}=360\pi x\frac{\mathrm{d}x}{\mathrm{dt}}-\pi x^2\frac{\mathrm{d}x}{\mathrm{dt}}$$

$$\left.\frac{\mathrm{dV}}{\mathrm{dt}}\right|_{x=120}=\frac{\mathrm{d}x}{\mathrm{dt}}\left(360\pi\times 120-120^2\pi\right)$$

$$\therefore \quad 1800=\frac{dx}{dt}\left(360\pi\times 120-120^2\pi\right)$$

$$\therefore \quad 15=\frac{dx}{dt}(360\pi-120\pi)$$

$$\frac{\mathrm{d}x}{\mathrm{dt}}=\frac{15}{240\pi}=\frac{1}{16\pi}\ \mathrm{cm}/\mathrm{sec}$$

Question92

A point moves along the arc of parabola $y=2x^2$. Its abscissa increases uniformly at the rate of 2 units $/$sec. At the instant, the point is passing through $(1,2)$, its distance from origin is increasing at the rate of MHT CET 2024 (10 May Shift 1)

Options:

A. $\frac{36}{\sqrt{5}}$ units/sec.

B. $\frac{18}{\sqrt{5}}$ units /sec.

C. $\frac{36}{5}$ units/sec.

D. $\frac{18}{5}$ units / sec.

Answer: B

Solution:

Given, $\frac{dx}{dt} = 2$ units/sec

Given equation of parabola is $y = 2x^2$

Differentiating w.r.to t, we get

$$\frac{dy}{dt} = 4x \times \frac{dx}{dt}$$
$$\frac{dy}{dt} = 8x$$

$\ldots$ (i) $\left[\frac{dx}{dt} = 2\right]$

$\therefore$ The distance of point from origin is given by $\sqrt{x^2 + y^2}$

$\therefore$ The rate of increasing distance of point from origin

$$= \frac{d}{dt}\left(\sqrt{x^2 + y^2}\right)$$
$$= \frac{1}{2\sqrt{x^2 + y^2}} \cdot \frac{d}{dt}\left(x^2 + y^2\right)$$
$$= \frac{1}{2\sqrt{x^2 + y^2}} \cdot \left[2x \times \frac{dx}{dt} + 2y \times \frac{dy}{dt}\right]$$
$$= \frac{1}{2\sqrt{x^2 + y^2}} \cdot [4x + 2y \times 8x] \ldots [From(i)$$
$$= \frac{(2x + 8xy)}{\sqrt{x^2 + y^2}}$$

$\therefore$ Since point is passing through $(1, 2)$

$\therefore$ Rate of increasing distance of point from origin

$$= \frac{2(1) + 8(1)(2)}{\sqrt{1^2 + 2^2}}$$
$$= \frac{18}{\sqrt{5}} \text{ units /sec}$$

Question93

The equation of the normal to the curve $y = x \log x$, which is parallel to the line $2x - 2y + 3 = 0$, is MHT CET 2024 (10 May Shift 1)

Options:

A. $x + y = 3e^{-2}$

B. $x - y = 3e^{-2}$

C. $x - y = 3e^2$

D. $x + y = 3e^2$

Answer: B

Solution:

$y = x \log x$
Differentiating w.r.to x, we get

$\frac{dy}{dx} = 1 + \log x$

$\therefore$ Slope of tangent $= 1 + \log x$

Slope of given line $= 1$

$\therefore$ Line of normal is parallel to given line

$$\frac{dy}{dx} = -1$$
$$\Rightarrow 1 + \log x = -1$$
$$\Rightarrow \log x = -2$$
$$\Rightarrow x = e^{-2}$$

Substituting $x = e^{-2}$ in given equation,

$$y = x \log x$$
$$= e^{-2} \log e^{-2}$$
$$= -2e^{-2}$$

$\therefore$ Equation of line passing through $(e^{-2}, -2e^{-2})$ is

$$y + 2e^{-2} = 1(x - e^{-2})$$
$$\Rightarrow x - y = 3e^{-2}$$

Question94

The approximate value of $x^3 - 2x^2 + 3x + 2$ at $x = 2.01$ is MHT CET 2024 (10 May Shift 1)

Options:

A. 8.07

B. 8.27

C. 8.007

D. 8.17

Answer: A

Solution:

$f(x) = x^3 - 2x^2 + 3x + 2$
$f'(x) = 3x^2 - 4x + 3$
Here, $a = 2,\ h = 0.01$

$$\begin{aligned} f(a) &= (2)^3 - 2(2)^2 + 3(2) + 2 \\ &= 8 - 8 + 6 + 2 \\ &= 8 \end{aligned}$$

$$\begin{aligned} f'(a) &= 3(2)^2 - 4(2) + 3 \\ &= 12 - 8 + 3 \\ &= 7 \end{aligned}$$

$$\begin{aligned} \therefore \quad f(a+h) &= f(a) + hf'(a) \\ &= 8 + (0.01)7 \\ &= 8 + 0.07 \\ &= 8.07 \end{aligned}$$

Question95

The function $f(x) = 2x^3 - 6x + 5$ is an increasing function, if MHT CET 2024 (09 May Shift 2)

Options:

A. $0 < x < 1$

B. $-1 < x < 1$

C. $x < -1$ or $x > 1$

D. $-1 < x < -\frac{1}{2}$

Answer: C

Solution:

$$f(x) = 2x^3 - 6x + 5$$
$$\therefore \quad f'(x) = 6x^2 - 6$$

For $f(x)$ to be increasing, $f'(x) > 0$ $\Rightarrow 6x^2 - 6 > 0 \Rightarrow (x-1)(x+1) > 0$
$\Rightarrow x > 1$ or $x < -1$

Question96

A square plate is contracting at the uniform rate $3\ \text{cm}^2/\text{sec}$, then the rate at which the perimeter is decreasing, when the side of the square is 15 cm , is MHT CET 2024 (09 May Shift 2)

Options:

A. $\frac{1}{5}\ \text{cm}/\text{sec}$

B. $\frac{2}{5}\ \text{cm}/\text{sec}$

C. $\frac{1}{10}\ \text{cm}/\text{sec}$

D. $\frac{3}{10}\ \text{cm}/\text{sec}$

Answer: B

Solution:

Let A, P and X be the area, perimeter and length of side of square respectively at time ' t ' seconds.
Then,

$$\begin{aligned} & A=X^2, P=4X \\ \therefore\ & P=4\sqrt{A} \end{aligned}$$

Differentiating w.r.t. t, we get

$$\begin{aligned} \frac{\mathrm{dP}}{\mathrm{dt}} & =4 \frac{1}{2\sqrt{\mathrm{A}}} \cdot \frac{\mathrm{dA}}{\mathrm{dt}} \\ & =\frac{2}{\mathrm{X}} \cdot \frac{\mathrm{dA}}{\mathrm{dt}} \\ & =\frac{2}{15} \times 3 \\ & =\frac{2}{5} \mathrm{~cm/sec} \end{aligned}$$

$$\because \begin{aligned} & \text{side } =15 \mathrm{~cm} \\ & \tfrac{\mathrm{dA}}{\mathrm{dt}}=3 \mathrm{~cm}^2/\mathrm{sec} \end{aligned}$$

Question97

A poster is to be printed on a rectangular sheet of paper of area $18 \mathrm{~m}^2$. The margins at the top and bottom of 75 cm each and at the sides 50 cm each are to be left. Then the dimensions i.e. height and breadth of the sheet so that the space available for printing is maximum, are _________ respectively. MHT CET 2024 (09 May Shift 2)

Options:

A. $2\sqrt{3} \mathrm{~m}, 3\sqrt{3} \mathrm{~m}$

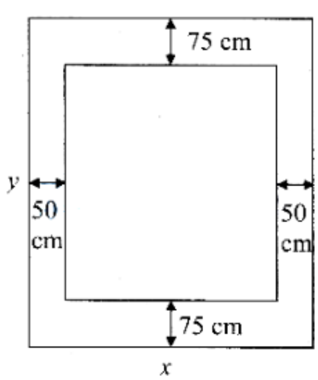
B. $3\sqrt{3} \mathrm{~m}, 2\sqrt{3} \mathrm{~m}$

C. $3 \mathrm{~m}, 6 \mathrm{~m}$

D. $6 \mathrm{~m}, 3 \mathrm{~m}$

Answer: B

Solution:



Let height and breadth of the sheet be ' y ' m and ' x ' m respectively.

$\therefore \quad xy = 180000 \text{ cm}^2$
$\therefore \quad y = \frac{180000}{x}$

$\therefore$ The area available for printing is

$$\begin{aligned} A &= (y-150)(x-100) \\ &= \left(\frac{180000}{x} - 150\right)(x-100) \\ &= 180000 - \frac{18000000}{x} - 150x - 15000 \\ &= 165000 - 150x - \frac{18000000}{x} \end{aligned}$$

$\therefore \quad \frac{\mathrm{dA}}{\mathrm{d}x} = 0 - 150 + \frac{18000000}{x^2}$
$\therefore \quad \frac{\mathrm{dA}}{\mathrm{d}x} = 0 \Rightarrow x^2 = \frac{18000000}{150} = 120000$
$\Rightarrow x = 200\sqrt{3} \text{ cm}$
$\Rightarrow y = \frac{180000}{200\sqrt{3}} = 300\sqrt{3} \text{ cm}$
Now, $\frac{\mathrm{d}^2 \mathrm{~A}}{\mathrm{d}x^2} = \frac{-36000000}{x^3}$

$\therefore$ At $x = 200\sqrt{3}$ cm, $\frac{\mathrm{d}^2 \mathrm{~A}}{\mathrm{d}x^2} < 0$

$\therefore$ Area is maximum at $x = 200\sqrt{3}$ cm and $y = 300\sqrt{3}$ cm

$\therefore \quad y = 3\sqrt{3}$ m and $x = 2\sqrt{3}$ m

Question98

The approximate value of $(3 \cdot 978)^{3/2}$ is MHT CET 2024 (09 May Shift 2)

Options:

A. 7.096

B. 8.096

C. 7.934

D. 8.934

Answer: C

Solution:

$$\text{Let } f(x) = x^{\frac{3}{2}}$$
$$\therefore \quad f'(x) = \frac{3}{2}x^{\frac{1}{2}}$$

Here, $a = 4$ and $h = -0.022$.

$$f(a) = f(4) = 4^{\frac{3}{2}} = 8$$
$$f'(a) = f'(4) = \frac{3}{2}(4)^{\frac{1}{2}} = 3$$
$$\therefore \quad f(a+h) \approx f(a) + hf'(a)$$
$$\approx 8 + (-0.022)(3)$$
$$\approx 8 - 0.066$$
$$\approx 7.934$$

Question99

The equation of the tangent to the curve $x = a\cos^3\theta, y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$ is MHT CET 2024 (09 May Shift 2)

Options:

A. $x + y = \frac{a}{\sqrt{2}}$

B. $x + y = \frac{a}{2}$

C. $x + y = \frac{a}{2\sqrt{2}}$

D. $x + y = \frac{a}{8}$

Answer: A

Solution:

$$x = a\cos^3\theta \text{ and } y = a\sin^3\theta$$
$$\therefore \quad \frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta \text{ and } \frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$$
$$\therefore \quad \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = -\tan\theta$$
$$\therefore \quad \left(\frac{dy}{dx}\right)_{\left(\theta=\frac{\pi}{4}\right)} = -1$$
$$\text{At } \theta = \frac{\pi}{4}$$
$$x = a\cos^3\frac{\pi}{4} = \frac{a}{2\sqrt{2}}$$
$$y = a\sin^3\frac{\pi}{4} = \frac{a}{2\sqrt{2}}$$

$\therefore$ Equation of the tangent at $\left(\frac{a}{2\sqrt{2}}, \frac{a}{2\sqrt{2}}\right)$ is

$$y - \frac{a}{2\sqrt{2}} = -1\left(x - \frac{a}{2\sqrt{2}}\right)$$
$$\Rightarrow x + y = \frac{a}{\sqrt{2}}$$

Question100

The population of a town increases at a rate proportional to the population at that time. If the population increases from 40 thousand to 80 thousand in 40 years, then the population in another 40

years will be MHT CET 2024 (09 May Shift 1)

Options:

A. 180000

B. 128000

C. 160000

D. 256000

Answer: C

Solution:

Let p be the population at time t years.

Then $\frac{dp}{dt} = kp$

$\Rightarrow \frac{dp}{p} = kdt$

Integrating on both sides, we get

$\log p = kt + c$

when $t = 0, p = 40000$

$\therefore \quad \log 40000 = 0 + c$

$\Rightarrow c = \log 40000$

$\therefore \quad \log p = kt + \log 40000$

$\Rightarrow \log\left(\frac{p}{40000}\right) = kt$

When $t = 40$ years, $p = 80000$

$\Rightarrow \log\left(\frac{80000}{40000}\right) = 40k$

$\Rightarrow k = \frac{1}{40}\log 2$

$\therefore \quad \log\left(\frac{p}{40000}\right) = \frac{1}{40}\log 2 \times t$

$\therefore$ Population after another 40 years, i.e., $t = 80$ years, we have

$\log\left(\frac{p}{40000}\right) = \frac{1}{40}\log 2 \times 80$

$\Rightarrow \log \frac{p}{40000} = 2\log 2$

$\Rightarrow \frac{p}{40000} = 4$

$\Rightarrow p = 16,0000$

Question101

Let C be a curve given by $y(x) = 1 + \sqrt{4x - 3}$, $x > \frac{3}{4}$. If P is a point on C , such that the tangent at P has slope $\frac{2}{3}$, then a point through which the normal at P passes, is MHT CET 2024 (09 May Shift 1)

Options:

A. $(1,7)$

B. $(3,-4)$

C. $(4,-3)$

D. $(2,3)$

Answer: A

Solution:

$$\begin{aligned} & y(x)=1+\sqrt{4x-3} \\ \therefore\quad & \frac{dy}{dx}=\frac{4}{2\sqrt{4x-3}}=\frac{2}{\sqrt{4x-3}} \\ & \Rightarrow \frac{2}{\sqrt{4x-3}}=\frac{2}{3} \\ & \Rightarrow x=3 \\ \therefore\quad & y=4 \end{aligned}$$

$\therefore$ Equation of normal is

$$\begin{aligned} & y-4=\frac{-3}{2}(x-3) \\ & \Rightarrow 2y-8=-3x+9 \\ & \Rightarrow 3x+2y-17=0 \end{aligned}$$

$\therefore$ Option (A) i.e., $(1,7)$ satisfies above equation.

Question102

If $f(x)=\frac{\log x}{x}(x>0)$, then it is increasing in MHT CET 2024 (09 May Shift 1)

Options:

A. $(0, e)$

B. (e, ∞)

C. $(0, \infty)$

D. $(-\infty, \infty)$

Answer: A

Solution:

$$\begin{aligned} & f(x)=\frac{\log x}{x} \\ \therefore\quad & f'(x)=\frac{1}{x^2}-\frac{\log x}{x^2}=\frac{1-\log x}{x^2} \end{aligned}$$

For $f(x)$ to be increasing, $f'(x)>0$

$$\Rightarrow 1-\log x>0 \Rightarrow 1>\log x \Rightarrow e>x$$

$\therefore$ $f(x)$ is increasing in the interval $(0, e)$.

Question103

The maximum value of $\frac{\log x}{x}$ is MHT CET 2024 (09 May Shift 1)

Options:

A. e

B. 2e

C. $\frac{1}{e}$

D. $\frac{2}{e}$

Answer: C

Solution:

Let $f(x) = \frac{\log x}{x} \Rightarrow f'(x) = \frac{1}{x^2} - \frac{\log x}{x^2}$

For maximum or minimum value of $f(x)$,

$$f'(x) = 0$$
$$\Rightarrow \frac{1 - \log_e x}{x^2} = 0$$

$\therefore \quad \log_e x = 1$ or $x = e$, which lie in $(0, \infty)$.

For $x = e$, $\frac{d^2y}{dx^2} = -\frac{1}{e^3}$, which is -ve .

$\therefore \quad y$ is maximum at $x = e$ and its maximum value $= \frac{\log e}{e} = \frac{1}{e}$.

Question104

If the curves $y^2 = 6x, 9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is MHT CET 2024 (09 May Shift 1)

Options:

A. $\frac{9}{2}$

B. 6

C. 7

D. $\frac{7}{2}$

Answer: A

Solution:

$y^2 = 6x..(i)$

$\Rightarrow 2y\frac{dy}{dx} = 6 \Rightarrow \frac{dy}{dx} = \frac{3}{y}$

Also, $9x^2 + by^2 = 16$

$\Rightarrow 18x + 2\,by\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-9x}{by}$

As given curves intersect each other at right angle, their tangents also intersect at right angles.

$$\Rightarrow \quad \begin{aligned} &\tfrac{3}{y} \times \tfrac{-9x}{by} = -1 \\ &\Rightarrow y^2 = 27x \\ &\text{(i)} \Rightarrow b(6x) = 27x \\ &\Rightarrow b = \tfrac{9}{2} \end{aligned}$$

Question105

If Mean value theorem holds for the function $f(x) = (x-1)(x-2)(x-3), x \in [0, 4]$ then the values of c as per the theorem are MHT CET 2024 (09 May Shift 1)

Options:

A. $2 \pm \frac{4}{\sqrt{3}}$

B. $2 \pm \frac{2}{\sqrt{3}}$

C. $2 \pm \sqrt{2}$

D. $2 \pm \sqrt{3}$

Answer: B

Solution:

$f(x) = (x-1)(x-2)(x-3)$

$f(4) = (4-1)(4-2)(4-3)$

$f(4) = 6$

$f(0) = (0-1)(0-2)(0-3) = -6$

$\therefore$ Using LMVT

$f'(c) = \frac{f(4)-f(0)}{4-0} = \frac{6-(-6)}{4} = 3$

$\therefore \quad f'(c) = 3...(i)$

Now, $f(x) = (x-1)(x-2)(x-3)$

$= x^3 - 6x^2 + 11x - 6$

$\therefore \quad f'(x) = 3x^2 - 12x + 11$

$f'(c) = 3$

$\Rightarrow 3c^2 - 12c + 11 = 3$

$\Rightarrow 3c^2 - 12c + 8 = 0$

$\Rightarrow c = \frac{12 \pm \sqrt{48}}{6} = 2 \pm \frac{2\sqrt{3}}{3} = 2 \pm \frac{2}{\sqrt{3}}$

...[From (i)]

Question106

The approximate value of $\tan^{-1}(0.999)$ is (use $\pi = 3.1415$) MHT CET 2024 (04 May Shift 2)

Options:

A. 0.7843

B. 0.7849

C. 0.7847

D. 0.7851

Answer: B

Solution:

$$\text{Let } f(x) = \tan^{-1} x$$

$$\therefore \quad f'(x) = \frac{1}{1+x^2}$$

Here, $a = 1$ and $h = -0.001$

$$\therefore \quad f(a+h) \approx f(a) + hf'(a)$$

$$\begin{aligned} \therefore \quad \tan^{-1}(0.999) &\approx \frac{\pi}{4} + \frac{1}{1+1}(-0.001) \\ &\approx \frac{\pi}{4} - \frac{0.001}{2} \\ &\approx \frac{\pi}{4} - 0.0005 \\ &\approx \frac{3.1415}{4} - 0.0005 \\ &\approx 0.7849 \end{aligned}$$

Question107

A ladder 5 m long rests against a vertical wall. If its top slides downwards at the rate of 10 cm/sec., then the foot of the ladder is sliding at the rate of _______ m/sec, when it is 4 m away from the wall. MHT CET 2024 (04 May Shift 2)

Options:

A. 0.75

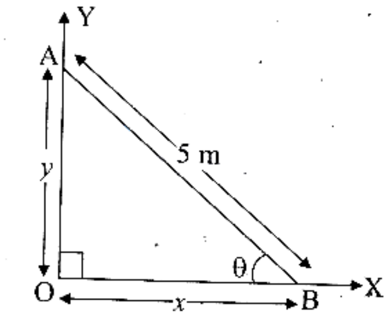
B. 7.5

C. 0.0075

D. 0.075

Answer: D

Solution:



According to the figure, $x^2 + y^2 = 25$

At $x = 4, y = 3$

Differentiating (i) with respect to ' t ', we get

$$2x\frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$$
$$x\frac{dx}{dt} = -y\frac{dy}{dt}$$
$$\frac{dx}{dt} = \frac{-y}{x}\frac{dy}{dt} = \frac{-3}{4} \times (-0.1) = 0.075$$

Question108

The equation of the tangent to the curve $y = 1 - e^{\frac{x}{3}}$ at the point of intersection with Y -axis is MHT CET 2024 (04 May Shift 2)

Options:

A. $x - 3y = 0$

B. $x + 3y = 0$

C. $x + 2y = 0$

D. $3x' + y = 0$

Answer: B

Solution:

Given equation of curve is

$y = 1 - e^{\frac{x}{3}} \ldots (i)$

Since, curve intersects Y-axis, $x = 0$

$$\therefore \quad y = 1 - e^{\frac{0}{3}} = 1 - 1$$
$$\Rightarrow y = 0$$

$\therefore$ Tangent to the curve passes through origin

$\therefore$ Slope of tangent $= \frac{dy}{dx}$

$\therefore$ Differentiating (i) w.r.to x, we get

$$\frac{dy}{dx} = \frac{-e^{\frac{x^3}{3}}}{3}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(0,0)} = \frac{-e^{\frac{0}{3}}}{3} = \frac{-1}{3}$$

$\therefore$ Equation of tangent is

$$y - 0 = \frac{-1}{3}(x - 0)$$
$$\Rightarrow 3y = -x$$
$$\Rightarrow x + 3u = 0$$

Question109

If $f(x) = \left(\sin^4 x + \cos^4 x\right), 0 < x < \frac{\pi}{2}$, then the function has minimum value _______ at $x =$ ________. MHT CET 2024 (04 May Shift 2)

Options:

A. $0.7934, \frac{\pi}{9}$

B. $\frac{1}{2}, \frac{\pi}{4}$

C. $\frac{5}{8}, \frac{\pi}{3}$

D. $0.75, \frac{\pi}{8}$

Answer: B

Solution:

$$
\begin{aligned}
f(x) &= \sin^4 x + \cos^4 x \\
&= \left(\sin^2 x + \cos^2 x\right)^2 - 2\sin^2 x \cos^2 x \\
&= 1 - \frac{1}{2}(\sin 2x)^2
\end{aligned}
$$

Since $0 \leq \sin^2 2x \leq 1$

$$\therefore \quad 0 \geq -\tfrac{1}{2}\sin^2 2x \geq -\tfrac{1}{2}$$

$$
\begin{aligned}
&\Rightarrow 1 + 0 \geq 1 - \frac{1}{2}\sin^2 2x \geq 1 - \frac{1}{2} \\
&\Rightarrow 1 \geq \sin^4 x + \cos^4 x \geq \frac{1}{2}
\end{aligned}
$$

$$
\begin{aligned}
&\Rightarrow 1 - \frac{1}{2}(\sin 2x)^2 = \frac{1}{2} \\
&\Rightarrow (\sin 2x)^2 = 1 \\
&\Rightarrow (\sin 2x)^2 = \left(\sin \frac{\pi}{2}\right)^2 \\
&\Rightarrow 2x = \frac{\pi}{2} \\
&\Rightarrow x = \frac{\pi}{4}
\end{aligned}
$$

Question110

The function $f(x) = \frac{\log_e(\pi+x)}{\log_e(e+x)}$ is MHT CET 2024 (04 May Shift 1)

Options:

A. increasing on $(0, \infty)$.

B. increasing on $\left(0, \frac{\pi}{e}\right)$, decreasing on $\left(\frac{\pi}{e}, \infty\right)$.

C. decreasing on $(0, \infty)$.

D. decreasing on $\left(0, \frac{\pi}{e}\right)$, increasing on $\left(\frac{\pi}{e}, \infty\right)$

Answer: C

Solution:

$$
\begin{aligned}
&\text{Let } \mathrm{f}(x) = \frac{\ln(\pi + x)}{\ln(\mathrm{e} + x)} \\
&\therefore \quad \mathrm{f}'(x) = \frac{\ln(\mathrm{e} + x) \times \frac{1}{\pi + x} - \ln(\pi + x) \times \frac{1}{\mathrm{e} + x}}{[\ln(\mathrm{e} + x)]^2} \\
&\qquad = \frac{(\mathrm{e} + x)\ln(\mathrm{e} + x) - (\pi + x)\ln(\pi + x)}{[\ln(\mathrm{e} + x)]^2 \times (\mathrm{e} + x)(\pi + x)} \\
&\Rightarrow \quad \Rightarrow f'(x) < 0 \text{ for all } x > 0 \\
&\quad \therefore \quad \mathrm{f}(x) \text{ is decreasing on } (0, \infty).
\end{aligned}
$$

Question111

The sum of intercepts on coordinate axes made by tangent to the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ is MHT CET 2024 (04 May Shift 1)

Options:

A. a

B. 2 a

C. $2\sqrt{a}$

D. $\sqrt{2}a$

Answer: A

Solution:

$\sqrt{x}+\sqrt{y}=\sqrt{a}$

Differentiating w.r.t. x, we get

$$\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0$$

$\therefore$ Slope of the tangent is $\frac{dy}{dx}=-\sqrt{\frac{y}{x}}$

$\therefore$ Equation of the tangent at (x_1, y_1) is

$$(y-y_1)=-\sqrt{\frac{y_1}{x_1}}(x-x_1)$$

$$\sqrt{x_1}y+\sqrt{y_1}x=\sqrt{x_1}y_1+\sqrt{y_1}x_1$$

$$\therefore \sqrt{y_1}x+\sqrt{x_1}y=\sqrt{x_1y_1}\left(\sqrt{x_1}+\sqrt{y_1}\right)$$

$$\therefore \sqrt{y_1}x+\sqrt{x_1}y=\sqrt{x_1y_1a}$$

$$\therefore \frac{x}{\sqrt{x_1a}}+\frac{y}{\sqrt{y_1a}}=1$$

$$\therefore x\text{-intercept } + y\text{-intercept } =\sqrt{a}\left(\sqrt{x_1}+\sqrt{y_1}\right)$$

$$=\sqrt{a}\times\sqrt{a}=a$$

Question112

A wire of length 2 units is cut into two parts, which are bent respectively to form a square of side x units and a circle of radius of r units. If the sum of the areas of square and the circle so formed is minimum, then MHT CET 2024 (04 May Shift 1)

Options:

A. $2x=(\pi+4)r$

B. $(4-\pi)x=\pi r$

C. $x=2r$

D. $2x=r$

Answer: C

Solution:

Perimeter of the square $= 4x$

Perimeter of the circle $= 2\pi r$

$\therefore \quad 4x + 2\pi r = 2$
$\therefore \quad 2x + \pi r = 1 \Rightarrow r = \frac{1-2x}{\pi} \cdots (i)$
Sum of the areas $(A) = x^2 + \pi r^2$

$\therefore \quad A = x^2 + \pi\left(\frac{1-2x}{\pi}\right)^2$

...[From (i)

$\therefore \quad A = x^2 + \frac{1}{\pi}(1-2x)^2$

Differentiating A w.r.t. x, we get

$$\frac{dA}{dx} = 2x + \frac{2}{\pi}(1-2x)(-2), \frac{d^2 A}{dx^2} = 2 + \frac{8}{\pi} > 0$$
$$\frac{dA}{dx} = 0 \Rightarrow 2x - \frac{4}{\pi} + \frac{8x}{\pi} = 0$$
$$\Rightarrow (2\pi + 8)x = 4$$
$$\Rightarrow (\pi + 4)x = 2$$
$$\Rightarrow x = \frac{2}{\pi + 4}$$

$\therefore$ Area is minimum when $x = \frac{2}{\pi+4}$

Substituting $x = \frac{2}{\pi+4}$ in equation (i), we get

$$r = \frac{1}{\pi + 4}$$
$$\Rightarrow x = 2r$$

Question113

If sum of two numbers is 3 , then the maximum value of the product of first number and square of the second number is MHT CET 2024 (04 May Shift 1)

Options:

A. 6

B. 4

C. 5

D. 3

Answer: B

Solution:

Let the two numbers be a and b .

$\therefore \quad a+b=3$
$\therefore \quad b=3-a \ldots(i)$

$\therefore$ Product of first number and square of second number $(p)=a b^2$

$\mathrm{p}=\mathrm{a}(3-\mathrm{a})^2$

...[From (i)]

$\therefore \quad p=9 a-6 a^2+a^3 \ldots(i i)$
$\therefore \quad \frac{d p}{d a}=9-12 a+3 a^2$
$\therefore \quad \frac{d^2 p}{d a^2}=-12+6 a$
Now, $\frac{\mathrm{dp}}{\mathrm{da}}=0 \Rightarrow 3 \mathrm{a}^2-12 \mathrm{a}+9=0$

$\Rightarrow(3 a-9)(a-1)=0$
$\Rightarrow a=3$ or $a=1$

$\left.\frac{\mathrm{d}^2 p}{\mathrm{dda}^2}\right|_{a=3}=6>0$ and $\left.\frac{\mathrm{d}^2 p}{\mathrm{da}^2}\right|_{\mathrm{a}=1}=-6<0$

$\therefore$ Maximum value of p is at $\mathrm{a}=1$

$\therefore$ Maximum value of the product $=4$

...[From (ii)]

Question114

A wet substance in the open air loses its moisture at a rate proportional to the moisture content. If a sheet hung in the open air loses half its moisture during the first hour, then the time t , in which 99% of the moisture will be lost, is MHT CET 2024 (04 May Shift 1)

Options:

A. $\frac{2 \log 10}{\log 2}$

B. $\frac{\log 10}{\log 2}$

C. $\frac{3 \log 10}{\log 2}$

D. $\frac{1}{2} \frac{\log 10}{\log 2}$

Answer: A

Solution:

Let y be the amount of moisture at time t .

$$\therefore \quad \frac{dy}{dt} = -\alpha y$$
$$\Rightarrow \frac{dy}{y} = -\alpha dt$$

Integrating on both sides, we get

$\int \frac{dy}{y} = -\alpha \int dt$

$\therefore \quad \log y = -\alpha t + c$

when $t = 0, y = 1$

$\therefore$ From (i), we get

$c = 0$...(i)

when $t = 1, y = 0.5$

$\therefore$ From (i) and (ii), we get

$$\log(0.5) = -\alpha$$
$$\therefore \quad \alpha = \log 2$$

$\therefore$ From (i), (ii) and (iii), we get

$\log y = -(\log 2)t$

$\therefore$ When 99% of the moisture will be lost,

$$y = 0.01$$
$$\therefore \quad \log(0.01) = -(\log 2)t$$
$$\therefore \quad t = \frac{2 \log 10}{\log 2}$$

Question115

If $f(x) = x^3 + bx^2 + cx + d$ and $0 < b^2 < c$, then in $(-\infty, \infty)$ MHT CET 2024 (03 May Shift 2)

Options:

A. $f(x)$ is strictly increasing function

B. $f(x)$ is bounded

C. $f(x)$ has a local maxima

D. $f(x)$ is a strictly decreasing function

Answer: A

Solution:

$$f(x)=x^3+bx^2+cx+d$$
$$f'(x)=3x^2+2bx+c$$

$\therefore$

Now its discriminant $=4\left(b^2-3c\right)$

$\Rightarrow 4\left(b^2-c\right)-8c<0$, as $b^2<c$ and $c>0$

$\Rightarrow f'(x)>0$ for all $x \in R$

$\Rightarrow f$ is strictly increasing on R.

Question116

The minimum value of the function $f(x)=2x^3-15x^2+36x-48$ on the set $A=\left\{x \mid x^2+20 \leqslant 9x\right\}$ is MHT CET 2024 (03 May Shift 2)

Options:

A. -16

B. -7

C. 16

D. 7

Answer: A

Solution:

$$\begin{aligned} A &= \left\{x \mid x^2+20 \leq 9x\right\} \\ &= \left\{x \mid x^2-9x+20 \leq 0\right\} \\ &= \{x \mid (x-4)(x-5) \leq 0\} \\ A &= \{4,5\} \\ f(x) &= 2x^3-15x^2+36x-48 \\ \therefore \quad f'(x) &= 6x^2-30x+36 \\ &= 6\left(x^2-5x+6\right) \\ &= 6(x-2)(x-3)<0 \forall x \in (4,5) \end{aligned}$$

$\therefore$ $f(x)$ is strictly increasing in the interval $(4,5)$.

$\therefore$ Minimum value of $f(x)$ when $x \in (4,5)$ is

$f(4)=-16$

Question117

The value of c for which Rolle's theorem for the function $f(x)=x^3-3x^2+2x$ in the interval $[0,2]$ are MHT CET 2024 (03 May Shift 2)

Options:

A. ± 1

B. ± 2

C. $1 \pm \frac{1}{\sqrt{3}}$

D. $\sqrt{3}(1 \pm \sqrt{3})$

Answer: C

Solution:

$$
\begin{aligned}
& \mathrm{f}(x)=x^3-3 x^2+2 x \\
& \mathrm{f}^{\prime}(x)=3 x^2-6 x+2
\end{aligned}
$$

$$
\begin{aligned}
& \text{Now, } \mathrm{f}^{\prime}(\mathrm{c})=0 \\
& \Rightarrow 3 \mathrm{c}^2-6 \mathrm{c}+2=0 \\
& \Rightarrow \mathrm{c}=\frac{6 \pm \sqrt{12}}{6} \\
& \Rightarrow \mathrm{c}=1 \pm \frac{\sqrt{12}}{6} \\
& \Rightarrow \mathrm{c}=1 \pm \frac{1}{\sqrt{3}}
\end{aligned}
$$

Question118

If a body cools from $80^{\circ}\mathrm{C}$ to $60^{\circ}\mathrm{C}$ in the room temperature of $30^{\circ}\mathrm{C}$ in 30 min , then the temperature of a body after one hour is MHT CET 2024 (03 May Shift 2)

Options:

A. $42^{\circ}\mathrm{C}$

B. $24^{\circ}\mathrm{C}$

C. $48^{\circ}\mathrm{C}$

D. $56^{\circ}\mathrm{C}$

Answer: C

Solution:

Let θ be the temperature of the body at any time ' t '.

$\therefore \quad \frac{d\theta}{dt} \propto (\theta - 30)$

$\therefore \quad \frac{d\theta}{dt} = k(\theta - 30)$

Integrating on both sides, we get

$$\log(\theta - 30) = kt + C$$

when $t = 0, \theta = 80^\circ C$

$\therefore \quad \log(80 - 30) = k(0) + C$

$\Rightarrow \quad C = \log 50$

$\therefore \quad \log(\theta - 30) = kt + \log 50 \ldots (i)$

When $t = 30, \theta = 60$

$\therefore \quad \log 30 = 30k + \log 50$

$\Rightarrow \log 30 - \log 50 = 30k$

$\Rightarrow k = \frac{1}{30} \log\left(\frac{3}{5}\right)$

Equation (i) becomes,

$$\log(\theta - 30) = \frac{1}{30} \log\left(\frac{3}{5}\right) t + \log 50$$

when $t = 60$ minutes, we have

$$\log(\theta - 30) = \frac{1}{30} \log\left(\frac{3}{5}\right) \times 60 + \log 50$$

$$\log(\theta - 30) = 2 \log\left(\frac{3}{5}\right) + \log 50$$

$$\log(\theta - 30) = \log\left(\frac{9}{25} \times 50\right)$$

$\Rightarrow \theta - 30 = 18$

$\Rightarrow \theta = 48^\circ C$

Question119

A triangular park is enclosed on two sides by a fence and on the third side a straight river bank. The two sides having fence are of same length x.The maximum area (in sq. units) enclosed by the park is MHT CET 2024 (03 May Shift 1)

Options:

A. $\frac{3}{2} x^2$

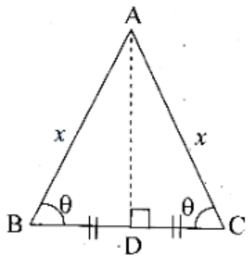
B. $\sqrt{\frac{x^3}{8}}$

C. $\frac{1}{2} x^2$

D. πx^2

Answer: C

Solution:



Let $\triangle \mathrm{ABC}$ be on isosceles triangle such that

$\mathrm{AB}=\mathrm{AC}=x$

$\therefore \quad \angle \mathrm{ABC}=\angle \mathrm{ACB}=\theta$

Draw seg $\mathrm{AD} \perp$ side BC at point D .

$\therefore \quad \triangle \mathrm{ABD}$ is a right angled triangle such that

$\mathrm{AD}=x \sin \theta$ and $\mathrm{BD}=x \cos \theta$

Similarly, in $\triangle \mathrm{ACD}$,

$\mathrm{DC}=x \cos \theta$

$\therefore \quad$ In $\triangle ABC$,

$$\begin{aligned} &\text{Height } =\mathrm{AD}=x \sin \theta \\ &\text{Base } =\mathrm{BC}=x \cos \theta+x \cos \theta=2 x \cos \theta \end{aligned}$$

$$\begin{aligned} \therefore \quad \mathrm{A}(\triangle \mathrm{ABC}) &=\frac{1}{2} \times x \sin \theta \times 2 x \cos \theta \\ &=\frac{x^2}{2}(2 \sin \theta \cos \theta) \\ &=\frac{x^2}{2} \sin 2 \theta \end{aligned}$$

Since, $-1 \leq \sin 2 \cdot \theta \leq 1$, for maximum value of $\sin 2\theta$, Maximum area $=\frac{x^2}{2}$ sq. units.

Question120

A stone is dropped into a quiet lake and waves move in circles at speed of $8 \mathrm{~cm} / \mathrm{sec}$. At the instant when the radius of the circular wave is 12 cm . how fast is the enclosed area increasing? MHT CET 2024 (03 May Shift 1)

Options:

A. $180 \pi \mathrm{~cm}^2 / \mathrm{sec}$

B. $196 \pi \mathrm{~cm}^2 / \mathrm{sec}$

C. $192 \pi \mathrm{~cm}^2 / \mathrm{sec}$

D. $200 \pi \mathrm{~cm}^2 / \mathrm{sec}$

Answer: C

Solution:

Given, the rate of increasing the radius

$$= \frac{dr}{dt} = 8 \text{ cm/sec}$$

$$\text{Area } = A = \pi r^2$$

$$\therefore \quad \frac{dA}{dt} = 2\pi \text{dr}$$

$$\Rightarrow \frac{dA}{dt} = 2\pi(12)(8)$$

$$\ldots [\because r = 12 \text{ cm}]$$

$$= 192\pi \text{ cm}^2/\text{sec}$$

Question121

A bullet is shot horizontally and its distance S cm at time t second is given by $S = 1200t - 15 \cdot t^2$, then the distance covered by the bullet when it comes to the rest, is MHT CET 2024 (03 May Shift 1)

Options:

A. 12000 cm

B. 24000 cm

C. 1200 cm

D. 2400 cm

Answer: B

Solution:

$s = 1200t - 15t^2$

$\text{Velocity}(v) = \frac{ds}{dt} = 1200 - 30t$

When bullet stopped, $\frac{ds}{dt} = 0$

$\Rightarrow t = 40$

Hence, required distance

$= 1200(40) - 15 \times (40)^2 = 24000 \text{ cm}$

Question122

The equation of the normal to the curve $x = \theta + \sin\theta, y = 1 + \cos\theta$ at $\theta = \frac{\pi}{2}$ is MHT CET 2024 (03 May Shift 1)

Options:

A. $2x + 2y + \pi = 0$

B. $2x - 2y - \pi = 0$

C. $x + y + \pi = 0$

D. $x + y - 2\pi = 0$

Answer: B

Solution:

$$x=\theta+\sin\theta, y=1+\cos\theta$$
$$\frac{dx}{d\theta}=1+\cos\theta, \frac{dy}{d\theta}=-\sin\theta$$
$$\therefore \quad \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{-\sin\theta}{1+\cos\theta}$$
$$\therefore \quad \left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}}=\frac{-\sin\left(\frac{\pi}{2}\right)}{1+\cos\left(\frac{\pi}{2}\right)}=\frac{-1}{1+0}=-1$$

$$\text{At } \theta=\frac{\pi}{2}$$
$$x=\frac{\pi}{2}+\sin\frac{\pi}{2}=1+\frac{\pi}{2}, y=1+\cos\frac{\pi}{2}=1$$

$\therefore$ Equation of the normal is

$$(y-1)=\frac{-1}{\left(\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}}\right)}\left(x-1-\frac{\pi}{2}\right)$$
$$\therefore \quad (y-1)=1\left(x-1-\frac{\pi}{2}\right)$$
$$\therefore \quad 2x-2y-\pi=0$$

Question123

The approximate value of $\cos\left(30^{\circ}, 30^{\prime}\right)$ is, given that $1^{\circ}=0.0175^{\circ}$ and $\cos 30^{\circ}=0.8660$ MHT CET 2024 (03 May Shift 1)

Options:

A. 0.8778

B. 0.7666

C. 0.7916

D. 0.8616

Answer: D

Solution:

$\cos(30°, 30')$

Let $f(x) = \cos x$

$\therefore\ f'(x) = -\sin x$

$\therefore\ f'(x) = -\sin x$

Now, $30°30' = 30° + 30'$

$= 30° + \left(\frac{1}{2}\right)^{\circ}$

$= 30° + \frac{0.0175}{2}$

$\therefore\ 30°30' = 30° + 0.00875$

Let $a = 30°$ and $h = 0.00875$

$\therefore\ \cos(30°, 30') = f(a+h)$

$\approx f(a) + hf'(a)$

$\approx f(30°) + 0.00875 f'(30°)$

$\approx \cos(30°) - 0.00875 \times \sin(30°)$

$\approx 0.8660 - 0.00875 \times 0.5$

≈ 0.8616

Question124

The co-ordinates of a point on the curve $y = x\log x$ at which the normal is parallel to the line $2x - 2y = 3$ are MHT CET 2024 (02 May Shift 2)

Options:

A. $\left(-e^{-2}, 2e^{-2}\right)$

B. $\left(-e^{-2}, -2e^{-2}\right)$

C. $\left(e^{-2}, 2e^{-2}\right)$

D. $\left(e^{-2}, -2e^{-2}\right)$

Answer: D

Solution:

$y = x\log x$

$\therefore\ \frac{dy}{dx} = 1 + \log x \ldots (i)$

Slope of the normal $= -\frac{1}{\left(\frac{dy}{dx}\right)} = \frac{-1}{1+\log x}$

Slope of the given line is 1 .

Since the normal is parallel to the given line.

$\therefore\ \frac{-1}{1+\log x} = 1$

$\Rightarrow \log x = -2$

$\Rightarrow x = e^{-2}$

From (i), $y = -2e^{-2}$

$\therefore$ Co-ordinates of the point are $\left(e^{-2}, -2e^{-2}\right)$.

Question125

The value of C for which Mean value Theorem holds for the function $\mathrm{f}(x)=\log_{\mathrm{e}} x$ on the interval $[1,3]$ is MHT CET 2024 (02 May Shift 2)

Options:

A. $\log_3 \mathrm{e}$

B. $\log_{\mathrm{e}} 3$

C. $\frac{1}{2}\log_{\mathrm{e}} 3$

D. $2\log_3 \mathrm{e}$

Answer: D

Solution:

$\mathrm{f}(x)=\log_{\mathrm{e}} x$
$\mathrm{f}(1)=\log_{\mathrm{e}} 1=0$
$\mathrm{f}(3)=\log_{\mathrm{e}} 3 \text{ and } \mathrm{f}'(x)=\frac{1}{x}$

By Lagrange's mean value theorem,

$$f'(c)=\frac{f(3)-f(1)}{3-1}$$
$$\Rightarrow \frac{1}{c}=\frac{\log_e 3-0}{2} \Rightarrow c=\frac{2}{\log_e 3} \Rightarrow \mathrm{c}=2\log_3 e$$

Question126

The maximum value of the function

$f(x)=3x^3-18x^2+27x-40$

on the set $\mathrm{S}=\left\{x\in\mathbb{R}/x^2+30\leq 11x\right\}$ is

MHT CET 2024 (02 May Shift 2)

Options:

A. -122

B. -222

C. 222

D. 122

Answer: D

Solution:

$S=\{x \in \mathbb{R} / x^2+30 \leq 11x\}$
$x^2+30 \leq 11x$
$\Rightarrow x^2-11x+30 \leq 0$
$\Rightarrow (x-5)(x-6) \leq 0$
$\Rightarrow x \in [5,6]$
Now, $f(x)=3x^3-18x^2+27x-40$

$f'(x)=9x^2-36x+27$
$f'(x)=9\left(x^2-4x+3\right)$
$=9\left[\left(x^2-4x+4\right)-1\right]$
$=9(x-2)^2-9$

$\therefore \quad f'(x)>0 \forall x \in [5,6]$

$\therefore \quad f(x)$ is strictly increasing in the interval $[5,6]$

$\therefore \quad$ Maximum value of $f(x)$ when $x \in [5,6]$ is

$f(6)=122$

Question127

The equation of normal to the curve $x=\theta+\sin\theta, y=1+\cos\theta$ at $\theta=\frac{\pi}{2}$ is MHT CET 2024 (02 May Shift 2)

Options:

A. $2x+2y-\pi=0$

B. $2x-y-\pi=0$

C. $2x-2y-\pi=0$

D. $2x+y-\pi=0$

Answer: C

Solution:

$x=\theta+\sin\theta$ and $y=1+\cos\theta$

$\therefore \quad \frac{dx}{d\theta}=1+\cos\theta$ and $\frac{dy}{d\theta}=-\sin\theta$

$\therefore \quad \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{-\sin\theta}{1+\cos\theta}$

At $\theta=\frac{\pi}{2}$

$x=\frac{\pi}{2}+\sin\frac{\pi}{2}=\frac{\pi}{2}+1$ and

$y=1+\cos\frac{\pi}{2}=1$

$\therefore \quad \left(\frac{dy}{dx}\right)_{\left(\theta=\frac{\pi}{2}\right)}=\frac{-\sin\frac{\pi}{2}}{1+\cos\frac{\pi}{2}}=-1$

$\therefore \quad$ Slope of normal $=1$

$\therefore \quad$ Equation of the normal at $\left(\frac{\pi}{2}+1,1\right)$ is

$y-1=1\left(x-\frac{\pi}{2}-1\right)$
$\Rightarrow 2y-2=2x-\pi-2$
$\Rightarrow 2x-2y-\pi=0$

Question128

The approximate value of $\sqrt[3]{0 \cdot 026}$ is MHT CET 2024 (02 May Shift 2)

Options:

A. 0.2762

B. 0.2963

C. 0.2632

D. 0.2692

Answer: B

Solution:

Let $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$

$\therefore \quad f'(x) = \frac{1}{3} x^{-\frac{2}{3}} = \frac{1}{3x^{\frac{2}{3}}}$

Here, $a = 0.027$ and $h = -0.001$

$\therefore \quad f(a) = f(0.027) = (0.027)^{\frac{1}{3}} = 0.3$ and

$f'(a) = f'(0.027)$

$= \frac{1}{3(0.027)^{\frac{2}{3}}}$

$= \frac{1}{3(0.09)} = \frac{1}{0.27}$

$\therefore \quad f(a+h) \approx f(a) + hf'(a)$

$\approx 0.3 - \frac{0.001}{0.27}$

$\approx 0.3 - 0.0037$

≈ 0.2963

Question129

A spherical metal ball at $80^{\circ}C$ cools in 5 minutes to $60^{\circ}C$, in surrounding temperature of $20^{\circ}C$, then the temperature of the ball after 20 minutes is approximately MHT CET 2024 (02 May Shift 1)

Options:

A. $(8.15)^{\circ}C$

B. $(11.85)^{\circ}C$

C. $(28.15)^{\circ}C$

D. $(31.85)^{\circ}C$

Answer: D

Solution:

Let θ be the temperature of ball at any time ' t '.

$$\therefore \quad \frac{d\theta}{dt} \propto (\theta - 20)$$
$$\Rightarrow \frac{d\theta}{dt} = -k(\theta - 20), k > 0$$

Integrating on both sides, we get

$$\log|\theta - 20| = -kt + c$$

when $t = 0, \theta = 80^\circ$

$$\therefore \quad c = \log 60$$
$$\therefore \quad \log|\theta - 20| = -kt + \log 60 \ldots(i)$$

$$\text{When } t = 5, \theta = 60^\circ$$
$$\therefore \quad \log 40 = -5k + \log 60$$
$$\Rightarrow 5k = \log 60 - \log 40$$
$$\Rightarrow 5k = \log\left(\frac{3}{2}\right)$$
$$\Rightarrow k = \frac{1}{5}\log\left(\frac{3}{2}\right)$$

$$\therefore \quad \log|\theta - 20| = \frac{-1}{5}\log\left(\frac{3}{2}\right)t + \log 60$$

...[From (i)]

when $t = 20$,

$$\Rightarrow \log|\theta - 20| = \frac{-1}{5}\log\left(\frac{3}{2}\right)20 + \log 60$$
$$\Rightarrow \log|\theta - 20| = -4\log\left(\frac{3}{2}\right) + \log 60$$
$$\Rightarrow \log|\theta - 20| = \log\left(\frac{2}{3}\right)^4 + \log 60$$
$$\Rightarrow \log|\theta - 20| = \log\left(\frac{16 \times 60}{81}\right)$$
$$\Rightarrow \log|\theta - 20| = \log(11.85)$$
$$\Rightarrow \theta - 20 = 11.85$$
$$\Rightarrow \theta = 11.85 + 20$$
$$\Rightarrow \theta = 31.85$$

The temperature of ball after 20 minutes is $31.85^\circ C$

Question130

If Rolle's theorem holds for the function $f(x) = x^3 + bx^2 + ax + 5$ on $[1, 3]$ with $c = 2 + \frac{1}{\sqrt{3}}$, then the values of a and b respectively are MHT CET 2024 (02 May Shift 1)

Options:

A. $-11, 6$

B. 11,6

C. $-11, -6$

D. $11, -6$

Answer: D

Solution:

Since $\mathrm{f}(x)$ satisfies the Rolle's theorem,

$$\mathrm{f}(1)=\mathrm{f}(3)$$
$$1+\mathrm{b}+\mathrm{a}+5=27+9\,\mathrm{b}+3\mathrm{a}+5$$
$$\Rightarrow\quad 2\mathrm{a}+8\,\mathrm{b}=-26$$
$$\Rightarrow \mathrm{a}+4\,\mathrm{b}=-13\ldots(i)$$

$$\mathrm{f}(x)=x^3+\mathrm{b}x^2+\mathrm{a}x+5$$
$$\therefore\quad \mathrm{f}'(x)=3x^2+2\,\mathrm{b}x+\mathrm{a}$$

Now, $f'(c)=0$

$$\Rightarrow \mathrm{f}'\left(2+\frac{1}{\sqrt{3}}\right)=0$$
$$\Rightarrow 3\left(2+\frac{1}{\sqrt{3}}\right)^2+2\,\mathrm{b}\left(2+\frac{1}{\sqrt{3}}\right)+\mathrm{a}=0$$
$$\Rightarrow 3\left(4+\frac{4}{\sqrt{3}}+\frac{1}{3}\right)+4\,\mathrm{b}+\frac{2\,\mathrm{b}}{\sqrt{3}}+\mathrm{a}=0$$
$$\Rightarrow \mathrm{a}+4\,\mathrm{b}+\frac{2\,\mathrm{b}+12}{\sqrt{3}}+13=0$$
$$\Rightarrow -13+\frac{2\,\mathrm{b}+12}{\sqrt{3}}+13=0\quad \ldots\text{[From (i)]}$$
$$\Rightarrow \frac{2\,\mathrm{b}+12}{\sqrt{3}}=0$$
$$\Rightarrow\ \mathrm{b}=-6$$

Substituting $b=-6$ in (i), we get $a=11$

Question131

The normal to the curve, $y(x-2)(x-3)=x+6$ at the point, where the curve intersects the Y-axis, passes through the point MHT CET 2024 (02 May Shift 1)

Options:

A. $\left(-\frac{1}{2},-\frac{1}{2}\right)$

B. $\left(\frac{1}{2},\frac{1}{2}\right)$

C. $\left(\frac{1}{2},-\frac{1}{3}\right)$

D. $\left(\frac{1}{2},\frac{1}{3}\right)$

Answer: B

Solution:

Given equation of curve

$$y(x-2)(x-3)=x+6$$
$$\Rightarrow y=\frac{x+6}{(x-2)(x-3)}$$
$$\Rightarrow y=\frac{x+6}{x^2-5x+6}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{\left(x^2-5x+6\right)(1)-(x+6)(2x-5)}{\left(x^2-5x+6\right)^2}$$
$$=\frac{x^2-5x+6-(x+6)(2x-5)}{\left(x^2-5x+6\right)^2}$$
$$\therefore \quad \frac{dy}{dx}=\frac{-x^2-12x+36}{\left(x^2-5x+6\right)^2}$$

At Y-axis, $x=0$

$$\therefore \quad \frac{dy}{dx_{\text{at} x=0}}=\frac{-(0)^2-12(0)+36}{\left(0^2-5(0)+6\right)^2}$$
$$=\frac{36}{36}$$
$$=1$$

The equation of normal is

$$y-1=-1(x-0)$$

i.e., $x+y=1$

Option (B) i.e.,

$\therefore \quad \left(\frac{1}{2},\frac{1}{2}\right)$ satisfies above equation

$\therefore \quad$ Normal passes through $\left(\frac{1}{2},\frac{1}{2}\right)$.

Question132

If the slope of the tangent of the curve at any point is equal to $-y+e^{-x}$, then the equation of the curve passing through origin is MHT CET 2023 (14 May Shift 2)

Options:

A. $y+xe^{x}=0$

B. $ye^{x}+x=0$

C. $ye^{x}-x=0$

D. $y-xe^{x}=0$

Answer: C

Solution:

$$\frac{dy}{dx} = -y + e^{-x}$$
$$\Rightarrow \frac{dy}{dx} + y = e^{-x}$$
$$\therefore \quad \text{I.F.} = e^{dx} = e^{x}$$

$\therefore$ Solution of the given equation is

$$ye^{x} = \int e^{x} \cdot e^{-x}\, dx + c$$
$$\Rightarrow ye^{x} = \int dx \pm c$$
$$\Rightarrow ye^{x} = x + c$$

Since the curve passes through $(0, 0)$.

$$\therefore \quad 0 = 0 + c$$
$$\Rightarrow c = 0$$
$$\therefore \quad ye^{x} = x$$
$$\Rightarrow ye^{x} - x^{2} = 0$$

Question133

The function $f(x) = x^3 - 6x^2 + 9x + 2$ has maximum value when x is MHT CET 2023 (14 May Shift 2)

Options:

A. 1

B. 2

C. 3

D. 6

Answer: A

Solution:

$$f(x) = x^3 - 6x^2 + 9x + 2$$
$$\therefore \quad f'(x) = 3x^2 - 12x + 9$$

For maximum or minimum,

$$f'(x) = 0$$
$$\Rightarrow 3x^2 - 12x + 9 = 0$$
$$\Rightarrow 3(x-1)(x-3) = 0$$
$$\Rightarrow x = 1, 3$$

Now, $f''(x) = 6x - 12$

$$\therefore \quad f''(1) = -6 < 0$$

$\therefore$ $f(x)$ is maximum at $x = 1$.

Question134

If $y = 4x - 5$ is a tangent to the curve $y^2 = \mathrm{p}x^3 + \mathrm{q}$ at $(2, 3)$, then $\mathrm{p} - \mathrm{q}$ is MHT CET 2023 (14 May Shift 2)

Options:

A. -5

B. 5

C. 9

D. -9

Answer: C

Solution:

$y^2 = \mathrm{p}x^3 + \mathrm{q} \ldots (i)$

Differentiating both sides w.r.t. x, we get

$$2y \cdot \frac{\mathrm{d}y}{\mathrm{d}x} = 3\mathrm{p}x^2$$
$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3\mathrm{p}}{2}\left(\frac{x^2}{y}\right)$$
$$\therefore \quad \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{(2,3)} = \frac{3\mathrm{p}}{2} \times \frac{4}{3} = 2\mathrm{p}$$

Slope of the line $y = 4x - 5$ is 4 .

Since the line touches the curve, their slopes are equal.

$\therefore \quad 2\mathrm{p} = 4 \Rightarrow \mathrm{p} = 2$

Since $(2, 3)$ lies on $y^2 = \mathrm{p}x^3 + \mathrm{q}$.

$$\therefore \quad 9 = 2 \times 8 + q \Rightarrow q = -7$$
$$\therefore \quad p - q = 2 + 7 = 9$$

Question135

The diagonal of a square is changing at the rate of $0.5\ \mathrm{cm/sec}$. Then the rate of change of area when the area is $400\ \mathrm{cm}^2$ is equal to MHT CET 2023 (14 May Shift 2)

Options:

A. $20\sqrt{2}\ \mathrm{cm}^2/\mathrm{sec}$

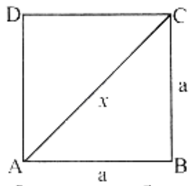
B. $10\sqrt{2}\ \mathrm{cm}^2/\mathrm{sec}$

C. $\frac{1}{10\sqrt{2}}\ \mathrm{cm}^2/\mathrm{sec}$

D. $\frac{10}{\sqrt{2}}\ \mathrm{cm}^2/\mathrm{sec}$

Answer: B

Solution:



$\frac{dx}{dt} = 0.5 \text{ cm/sec}$

$\therefore \quad \text{Area} = \frac{x^2}{2}$

$\therefore \quad \frac{dA}{dt} = \frac{2x}{2} \cdot \frac{dx}{dt} = x\frac{dx}{dt} = \frac{1}{2}x$

$$\therefore \quad \left[\frac{dA}{dt}\right]_{A=400} = \frac{1}{2}\sqrt{800} \quad \ldots \begin{bmatrix} \because A = 400 \text{ cm}^2 \\ x = \sqrt{800} \text{ cm} \end{bmatrix}$$
$$= 10\sqrt{2} \text{ cm}^2/\text{sec}$$

Question136

If $f(a) = 2, f'(a) = 1,\ g(a) = -1,\ g'(a) = 2$, then as x approaches a, $\frac{g(x)f(a)-g(a)f(x)}{(x-a)}$ approaches MHT CET 2023 (14 May Shift 2)

Options:

A. 3

B. 5

C. 0

D. 2

Answer: B

Solution:

Applying L-Hospital's rule, we get
$$\begin{aligned} &\lim_{x\to a} \frac{g(x)f(a)-g(a)f(x)}{(x-a)} \\ &= \lim_{x\to a} \frac{g'(x)f(a)-g(a)f'(x)}{1} \\ &= g'(a)f(a)-g(a)f'(a) \\ &= 2(2)-(-1)(1) \\ &= 4+1 \\ &= 5 \end{aligned}$$

Question137

Let $P(x)$ be a polynomial of degree 2 , with $P(2) = -1, P'(2) = 0, P''(2) = 2$, then $P(1.001)$ is MHT CET 2023 (14 May Shift 2)

Options:

A. 0.002

B. -0.002

C. 0.004

D. -0.004

Answer: B

Solution:

$$
\begin{aligned}
&\text{Let } \mathrm{P}(x) = \mathrm{a}x^2 + \mathrm{b}x + \mathrm{c} \\
&\Rightarrow \mathrm{P}'(x) = 2\mathrm{a}x + \mathrm{b} \\
&\Rightarrow \mathrm{P}''(x) = 2\mathrm{a} \\
&\mathrm{P}''(2) = 2\mathrm{a} \\
&\Rightarrow 2 = 2\mathrm{a} \\
&\Rightarrow \mathrm{a} = 1 \\
&\mathrm{P}'(2) = 2\mathrm{a}(2) + \mathrm{b} \\
&\Rightarrow 0 = 4\mathrm{a} + \mathrm{b} \\
&\Rightarrow 0 = 4(1) + \mathrm{b} \\
&\Rightarrow \mathrm{b} = -4 \\
&\mathrm{P}(2) = \mathrm{a}(2)^2 + \mathrm{b}(2) + \mathrm{c} \\
&\Rightarrow -1 = 4\mathrm{a} + 2\,\mathrm{b} + \mathrm{c} \\
&\Rightarrow -1 = 4(1) + 2(-4) + \mathrm{c} \\
&\Rightarrow \mathrm{c} = 3 \\
&\mathrm{P}(x) = x^2 - 4x + 3 \\
&\Rightarrow \mathrm{P}'(x) = 2x - 4 \\
&x = 1.001 = 1 + 0.001 = \mathrm{a} + \mathrm{h}
\end{aligned}
$$

Here, $\mathrm{a} = 1,\ \mathrm{h} = 0.001$

$$
\begin{aligned}
&\mathrm{P}(\mathrm{a}) = \mathrm{P}(1) = 1 - 4 + 3 = 0 \\
&\mathrm{P}'(\mathrm{a}) = \mathrm{P}'(1) = 2 - 4 = -2
\end{aligned}
$$

$\therefore \quad \mathrm{P}(1.001) = 0 + (0.001)(-2) = -0.002$

Question138

Let $\mathrm{f}(x) = 5 - |x - 2|$ and $\mathrm{g}(x) = |x + 1|, x \in \mathrm{R}$ If $\mathrm{f}(x)$ attains maximum value at α and $\mathrm{g}(x)$ attains minimum value at β, then $\lim_{x \to -\alpha\beta} \frac{(x-1)(x^2-5x+6)}{x^2-6x+8}$ is equal to MHT CET 2023 (14 May Shift 2)

Options:

A. $\frac{1}{2}$

B. $\frac{-3}{2}$

C. $\frac{-1}{2}$

D. $\frac{3}{2}$

Answer: A

Solution:

$$|x-2| \geq 0$$
$$\Rightarrow -|x-2| \leq 0$$
$$\Rightarrow 5-|x-2| \leq 5$$

Maximum value of $f(x)$ is 5 .

$$\therefore \quad 5-|x-2|=5$$
$$\Rightarrow |x-2|=0$$
$$\Rightarrow x=2$$
$$\Rightarrow \alpha=2$$
$$|x+1| \geq 0$$

Minimum value of $g(x)$ is 0 .

$$\therefore \quad |x+1|=0$$
$$\Rightarrow x=-1$$
$$\Rightarrow \beta=-1$$

$$\therefore \lim_{x \rightarrow -\alpha\beta} \frac{(x-1)\left(x^2-5x+6\right)}{x^2-6x+8}$$
$$=\lim_{x \rightarrow 2} \frac{(x-1)(x-2)(x-3)}{(x-2)(x-4)}$$
$$=\lim_{x \rightarrow 2} \frac{(x-1)(x-3)}{x-4}$$
$$=\frac{(1)(-1)}{-2}$$
$$=\frac{1}{2}$$

Question139

Let the curve be represented by $x=2(\cos t+t \sin t), y=2(\sin t-t \cos t)$. Then normal at any point ' t ' of the curve is at a distance of _____ units from the origin. MHT CET 2023 (14 May Shift 1)

Options:

A. 1

B. 0

C. 2

D. 4

Answer: C

Solution:

$$x = 2(\cos t + t \sin t)$$

$$\therefore \quad \frac{dx}{dt} = 2(-\sin t + \sin t + t \cos t) = 2t \cos t$$

$$y = 2(\sin t - t \cos t)$$

$$\therefore \quad \frac{dy}{dt} = 2(\cos t - \cos t + t \sin t) = 2t \sin t$$

$$\therefore \quad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t \sin t}{2t \cos t} = \tan t$$

$$\text{Slope of normal } = -\frac{1}{\frac{dy}{dx}} = -\frac{1}{\tan t} = -\frac{\cos t}{\sin t}$$

∴ Equation of the normal is

$$y - 2(\sin t - t\cos t) = -\frac{\cos t}{\sin t}[x - 2(\cos t + t \sin t)]$$

$$\Rightarrow y \sin t - 2\sin^2 t + 2t \sin t \cos t$$

$$= -x \cos t + 2\cos^2 t + 2t \sin t \cos t$$

$$\Rightarrow x \cos t + y \sin t = 2\left(\sin^2 t + \cos^2 t\right)$$

$$\Rightarrow x \cos t + y \sin t = 2$$

$$\therefore \quad \text{Distance from origin } = \left|\frac{-2}{\sqrt{\cos^2 t + \sin^2 t}}\right| = 2 \text{ units}$$

Question140

Let $B \equiv (0, 3)$ and $C \equiv (4, 0)$. The point A is moving on the line $y = 2x$ at the rate of 2 units/second. The area of $\triangle \mathrm{ABC}$ is increasing at the rate of MHT CET 2023 (14 May Shift 1)

Options:

A. $\frac{11}{\sqrt{5}}$ (units $)^2/$sec

B. $\frac{11}{5}$ (units) $)^2/$sec

C. $\frac{43}{\sqrt{5}}$ (units $)^2/$sec

D. $\frac{13}{5}$ (units) $)^2/$sec

Answer: A

Solution:

Let $A=(h, 2h)$

$$OA=\sqrt{h^2+4h^2}=\sqrt{5}h$$

$$\therefore \quad \frac{d(OA)}{dt}=\sqrt{5}\frac{dh}{dt}$$

$$\Rightarrow 2=\sqrt{5}\frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt}=\frac{2}{\sqrt{5}}$$

$$\begin{aligned} \alpha=A(\triangle ABC) &= \frac{1}{2}\begin{vmatrix} h & 2h & 1 \\ 0 & 3 & 1 \\ 4 & 0 & 1 \end{vmatrix} \\ &= \frac{1}{2}(3h+8h-12) \\ &= \frac{11h-12}{2} \\ \therefore \quad \frac{d\alpha}{dt} &= \frac{11}{2}\cdot\frac{dh}{dt} \\ &= \frac{11}{2}\cdot\frac{2}{\sqrt{5}} \\ &= \frac{11}{\sqrt{5}}(\text{ units })^2/\text{sec} \end{aligned}$$

Question141

The maximum value of the function $f(x)=3x^3-18x^2+27x-40$ on the set $S=\{x\in R/x^2+30\leq 11x\}$ is MHT CET 2023 (14 May Shift 1)

Options:

A. 122

B. -122

C. -222

D. 222

Answer: A

Solution:

$$\begin{aligned} S &= \{x\in R/x^2+30\leq 11x\} \\ &= \{x\in R/x^2-11x+30\leq 0\} \\ &= \{x\in R/(x-5)(x-6)\leq 0\} \\ &= \{x\in R/x\in[5,6]\} \\ f(x) &= 3x^3-18x^2+27x-40 \\ \therefore \quad f'(x) &= 9x^2-36x+27 \\ &= 9(x-1)(x-3)>0 \quad \forall x\in[5,6] \end{aligned}$$

$\Rightarrow f(x)$ is increasing in $[5,6]$.

$$\therefore \quad \text{Maximum value} = f(6) = 3(6)^3 - 18(6)^2 + 27(6) - 40 = 122$$

Question142

Let $f(x) = \int \frac{x^2-3x+2}{x^4+1} dx$, then function decreases in the interval MHT CET 2023 (14 May Shift 1)

Options:

A. $(-\infty, -2)$

B. $(-2, -1)$

C. $(1, 2)$

D. $(2, \infty)$

Answer: C

Solution:

$$f(x) = \int \frac{x^2 - 3x + 2}{x^4 + 1} dx$$
$$\Rightarrow f'(x) = \frac{x^2 - 3x + 2}{x^4 + 1}$$

For $f(x)$ to be decreasing,

$$f'(x) < 0$$
$$\Rightarrow \frac{x^2 - 3x + 2}{x^4 + 1} < 0$$
$$\Rightarrow \frac{(x-1)(x-2)}{x^4 + 1} < 0$$
$$\Rightarrow (x-1)(x-2) < 0$$
$$\Rightarrow x \in (1, 2)$$

Question143

Water is running in a hemispherical bowl of radius 180 cm at the rate of 108 cubic decimetres per minute. How fast the water level is rising when depth of the water level in the bowl is 120 cm ?(1 decimeter $= 10$ cm) MHT CET 2023 (13 May Shift 2)

Options:

A. $16\pi \text{cm/sec}$

B. $\frac{16}{\pi} \text{cm/sec}$

C. $\frac{1}{16\pi} \text{cm/sec}$

D. $\frac{\pi}{16}$ cm/sec

Answer: C

Solution:

Radius of hemispherical bowl $(\mathrm{r}) = 180\ \mathrm{cm}$ Rate of flow $\left(\frac{dV}{dt}\right) = 108 \mathrm{dm}^3/\mathrm{min}$

$$= 108000\ \mathrm{cm}^3/\mathrm{min}$$
$$= \frac{108000}{60}\ \mathrm{cm}^3/\mathrm{sec}$$
$$= 1800\ \mathrm{cm}^3/\mathrm{sec}$$

Let depth of water in bowl be x.

$\therefore$ Volume of water in hemispherical.

$$\text{bowl } (V) = \frac{\pi}{3} x^2 (3r - x)$$
$$V = \frac{\pi}{3} x^2 (3 \times 180 - x)$$

$$\therefore \quad V = 180 \pi x^2 - \frac{\pi}{3} x^3$$

Differentiating w.r.t. x, we get

$$\frac{dV}{dt} = 360 \pi x \frac{dx}{dt} - \pi x^2 \frac{dx}{dt}$$

$$\left.\frac{dV}{dt}\right|_{x=120} = \frac{dx}{dt}\left(360\pi \times 120 - 120^2 \pi\right)$$

$$\therefore \quad 1800 = \frac{dx}{dt}\left(360\pi \times 120 - 120^2 \pi\right)$$

$$\therefore \quad 15 = \frac{dx}{dt}(360\pi - 120\pi)$$

$$\frac{dx}{dt} = \frac{15}{240\pi}$$

$$= \frac{1}{16\pi} \mathrm{cm/sec}$$

Question144

$A(1,-3), B(4,3)$ are two points on the curve $y = x - \frac{4}{x}$. The points on the curve, the tangents at which are parallel to the chord AB, are MHT CET 2023 (13 May Shift 2)

Options:

A. $(1,2),(-1,-2)$

B. $(2,0),(-2,0)$

C. $(0,2),(1,-2)$

D. $(3,2),(-3,1)$

Answer: B

Solution:

Slope of tangent = slope of $\mathrm{AB}=\frac{3+3}{4-1}=\frac{6}{3}=2$

$y=x-\frac{4}{x}$

$\therefore \quad \frac{\mathrm{dy}}{\mathrm{d}x}=1+\frac{4}{x^2}$

$$\begin{aligned}&\Rightarrow 2=1+\frac{4}{x^2}\\&\Rightarrow x^2=4\\&\Rightarrow x=\pm 2\end{aligned}$$

When $x=2, y=2-\frac{4}{2}=0$

When $x=-2, y=-2+\frac{4}{2}=0$

$\therefore$ The required points are $(2,0)$ and $(-2,0)$.

Question145

Slope of the tangent to the curve $y=2\mathrm{e}^x \sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)$ where $0 \leq x \leq 2\pi$ is minimum at $x=$ MHT CET 2023 (13 May Shift 2)

Options:

A. 0

B. π

C. 2π

D. 1

Answer: B

Solution:

$y=2e^x \sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)$

$=e^x \sin\left(\frac{\pi}{2}-x\right)\ldots[2\sin\theta\cos\theta=\sin 2\theta]$

$=e^x \cos x$

$\therefore \quad \frac{dy}{dx}=e^x(\cos x-\sin x)$

Let $T=e^x(\cos x-\sin x)$

$\therefore \quad \frac{dT}{dx}=e^x(\cos x-\sin x)+e^x(-\sin x-\cos x)$

$=-2e^x \sin x$

Now, $\frac{dT}{dx}=0$

$\Rightarrow -2e^x \sin x=0$

$\Rightarrow \sin x=0$

$\Rightarrow x=0,\pi,2\pi \quad \ldots[\because 0\leq x\leq 2\pi]$

At $x=0,\ T=e^0(\cos 0-\sin 0)=1$

At $x=\pi, T=e^\pi(\cos\pi-\sin\pi)=-e^\pi$

At $x=2\pi, T=e^{2\pi}(\cos 2\pi-\sin 2\pi)=e^{2\pi}$

$\therefore$ Slope of the tangent is minimum at $x=\pi$

Question146

If Rolle's theorem holds for the function $f(x)=x^3+bx^2+ax+5$ on $[1,3]$ with $c=2+\frac{1}{\sqrt{3}}$, then the values of a and b respectively are MHT CET 2023 (13 May Shift 2)

Options:

A. $-11,-6$

B. 11,6

C. $11,-6$

D. 6,11

Answer: C

Solution:

Since $f(x)$ satisfies the Rolle's theorem,

$f(1) = f(3)$

$\therefore \quad 1 + b + a + 5 = 27 + 9b + 3a + 5$

$\Rightarrow 2a + 8b = -26$

$\Rightarrow a + 4b = -13$ ...(i)

$\vec{f}(x) = x^3 + bx^2 + ax + 5$

$\therefore \quad f'(x) = 3x^2 + 2bx + a$

Now, $f'(c) = 0$

$$\Rightarrow f'\left(2 + \frac{1}{\sqrt{3}}\right) = 0$$
$$\Rightarrow 3\left(2 + \frac{1}{\sqrt{3}}\right)^2 + 2b\left(2 + \frac{1}{\sqrt{3}}\right) + a = 0$$
$$\Rightarrow 3\left(4 + \frac{4}{\sqrt{3}} + \frac{1}{3}\right) + 4b + \frac{2b}{\sqrt{3}} + a = 0$$
$$\Rightarrow a + 4b + \frac{2b + 12}{\sqrt{3}} + 13 = 0$$
$$\Rightarrow -13 + \frac{2b + 12}{\sqrt{3}} + 13 = 0$$
$$\Rightarrow \frac{2b + 12}{\sqrt{3}} = 0$$
$$\Rightarrow b = -6$$

Substituting b = −6 in (i), we get a = 11

Question147

If slope of the tangent to the curve $xy + ax + by = 0$ at the point $(1, 1)$ on it is 2 , then the value of $3a + b$ is(MHT CET 2023 (13 May Shift 2)

Options:

A. 3

B. 1

C. 2

D. -1

Answer: B

Solution:

$xy + ax + by = 0$

Differentiating w.r.t. x, we get

$\frac{x\,dy}{dx} + y + a + b\frac{dy}{dx} = 0$

$$\Rightarrow \frac{dy}{dx} = -\frac{a+y}{b+x}$$
$$\Rightarrow \left(\frac{dy}{dx}\right)_{(b,1)} = -\frac{a+1}{b+1}$$
$$\Rightarrow 2 = -\frac{a+1}{b+1}$$

$\Rightarrow a + 2b = -3$ ...(i)

Also, the point $(1, 1)$ lies on the curve

$xy + ax + by = 0$.

$\therefore \quad 1 + a + b = 0$

$\Rightarrow a + b = -1$ ...(ii)

Solving (i) and (ii), we get $a = 1, b = -2$

$\therefore \quad 3a + b = 1$

Question148

If $f(x) = x^3 + bx^2 + cx + d$ and $0 < b^2 < c$, then in $(-\infty, \infty)$ MHT CET 2023 (13 May Shift 1)

Options:

A. $f(x)$ has a local maxima.

B. $f(x)$ is strictly increasing function.

C. $f(x)$ is bounded.

D. $f(x)$ is strictly decreasing function.

Answer: B

Solution:

$$f(x) = x^3 + bx^2 + cx + d$$
$$\therefore \quad f'(x) = 3x^2 + 2\,bx + c$$

Now its discriminant $= 4\left(b^2 - 3c\right) \Rightarrow 4\left(b^2 - c\right) - 8c < 0$, as $b^2 < c$ and $c > 0$

$\Rightarrow f'(x) > 0$ for all $x \in \mathrm{R}$

$\Rightarrow f$ is strictly increasing on R.

Question149

The slope of the normal to the curve $x=\sqrt{t}$ and $y=\mathrm{t}-\frac{1}{\sqrt{\mathrm{t}}}$ at $\mathrm{t}=4$ is MHT CET 2023 (13 May Shift 1)

Options:

A. $\frac{-17}{4}$

B. $\frac{4}{17}$

C. $\frac{-4}{17}$

D. $\frac{17}{4}$

Answer: C

Solution:

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{\mathrm{d}y}{\mathrm{dt}}}{\frac{\mathrm{d}x}{\mathrm{dt}}}=\frac{1+\frac{1}{2\mathrm{t}^{\frac{3}{2}}}}{\frac{1}{2\sqrt{\mathrm{t}}}}=\frac{2\mathrm{t}^{\frac{3}{2}}+1}{\mathrm{t}}$$

$$\therefore \quad \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{\mathrm{t}=4}=\frac{2(4)^{\frac{3}{2}}+1}{4}=\frac{16+1}{4}=\frac{17}{4}$$

$\therefore \quad$ Slope of normal at $\mathrm{t}=4$ is $-\frac{1}{\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{t=4}}=-\frac{4}{17}$

Question150

Values of c as per Rolle's theorem for $f(x)=\sin x+\cos x+6$ on $[0,2\pi]$ are MHT CET 2023 (13 May Shift 1)

Options:

A. $\frac{\pi}{3}, \frac{5\pi}{3}$

B. $\frac{\pi}{6}, \frac{5\pi}{6}$

C. $\frac{\pi}{4}, \frac{5\pi}{4}$

D. $\frac{\pi}{4}, \frac{7\pi}{4}$

Answer: C

Solution:

$$\therefore \quad f(x)=\sin x+\cos x+6$$
$$f'(x)=\cos x-\sin x$$
$$\text{Now, } f'(c)=0$$

$\Rightarrow \cos c-\sin c=0$

$\Rightarrow \cos c=\sin c$

$\Rightarrow \tan c=1$

$\Rightarrow c=\frac{\pi}{4}, \frac{5\pi}{4} \quad \ldots[\because x \in[0,2\pi]]$

Question151

A right circular cone has height $9 \mathrm{~cm}$ and radius of base $5 \mathrm{~cm}$. It is inverted and water is poured into it. If at any instant, the water level rises at the rate $\frac{\pi}{\mathrm{A}} \mathrm{cm}/\mathrm{sec}$. where A is area of the water surface at that instant, then cone is completely filled in MHT CET 2023 (13 May Shift 1)

Options:

A. 70sec.

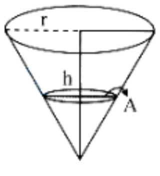
B. 75sec.

C. 72sec.

D. 77sec.

Answer: B

Solution:



For the conical vessel, $\mathrm{h}=9\ \mathrm{cm}, \mathrm{r}=5\ \mathrm{cm}$

$\therefore$ Full volume of the vessel,

$$\begin{aligned} \mathrm{V} &= \frac{1}{3}\pi\mathrm{r}^2\,\mathrm{h} \\ &= \frac{1}{3}\pi\times 25\times 9 \\ &= 75\pi\mathrm{cm}^3 \end{aligned}$$

Now, $\frac{\mathrm{h}}{\mathrm{r}}=\frac{9}{5}$

$$\begin{aligned} \therefore\quad & \mathrm{r}=\frac{5\,\mathrm{h}}{9} \\ \therefore\quad & \mathrm{A}=\pi\mathrm{r}^2=\pi\frac{25\,\mathrm{h}^2}{81} \end{aligned}$$

According to the given condition, $\frac{\mathrm{dh}}{\mathrm{dt}}=\frac{\pi}{\mathrm{A}}=\pi\frac{81}{\pi 25\,\mathrm{h}^2}=\frac{81}{25\,\mathrm{h}^2}$

$\therefore\quad h^2\mathrm{dh}=\frac{81}{25}\mathrm{dt}$

Integrating on both sides, we get

$$\begin{aligned} \frac{\mathrm{h}^3}{3} &= \frac{81}{25}\mathrm{t}+\mathrm{c}_1 \\ \therefore\quad \mathrm{h}^3 &= \frac{243}{25}\mathrm{t}+\mathrm{c},\ \text{where } \mathrm{c}=3\mathrm{c}_1 \end{aligned}$$

Naturally, $\mathrm{h}=0$, when $\mathrm{t}=0$ and hence, $\mathrm{c}=0$

$$\begin{aligned} \therefore\quad & \mathrm{h}^3=\frac{243}{25}\mathrm{t} \\ \therefore\quad & \mathrm{V}=\frac{1}{3}\pi\mathrm{r}^2\,\mathrm{h} \\ & =\frac{1}{3}\pi\frac{25\,\mathrm{h}^2}{81}\mathrm{h} \\ & =\frac{25}{243}\pi\mathrm{h}^3 \\ & =\frac{25}{243}\pi\frac{243}{25}\mathrm{t} \\ \therefore\quad & \mathrm{V}=\pi\mathrm{t} \end{aligned}$$

But volume of vessel, $\mathrm{V}=75\pi$

$$\begin{aligned} \therefore\quad & \pi\mathrm{t}=75\pi \\ \therefore\quad & \mathrm{t}=75 \text{ seconds.} \end{aligned}$$

Question152

A poster is to be printed on a rectangular sheet of paper of area $18\ \mathrm{m}^2$. The margins at the top and bottom of $75\ \mathrm{cm}$ each and at the sides $50\ \mathrm{cm}$ each are to be left. Then the dimensions i.e. height and

breadth of the sheet, so that the space available for printing is maximum, are respectively. MHT CET 2023 (12 May Shift 2)

Options:

A. $2\sqrt{3}$ m, $3\sqrt{3}$ m

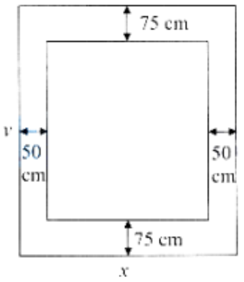
B. $3\sqrt{3}$ m, $2\sqrt{3}$ m

C. 3 m, 6 m

D. 6 m, 3 m

Answer: B

Solution:



Let height and breadth of the sheet be ' y ' m and ' x ' m respectively.

$$\therefore \quad xy = 180000 \text{ cm}^2$$
$$\therefore \quad y = \frac{180000}{x}$$

$\therefore$ The area available for printing is

$$\begin{aligned} A &= (y-150)(x-100) \\ &= \left(\frac{180000}{x} - 150\right)(x-100) \\ &= 180000 - \frac{18000000}{x} - 150x - 15000 \\ &= 165000 - 150x - \frac{18000000}{x} \end{aligned}$$
$$\therefore \frac{dA}{dx} = 0 - 150 + \frac{18000000}{x^2}$$
$$\therefore \frac{dA}{dx} = 0 \Rightarrow x^2 = \frac{18000000}{150} = 120000$$
$$\Rightarrow x = 200\sqrt{3} \text{ cm} \Rightarrow y = \frac{180000}{200\sqrt{3}} = 300\sqrt{3} \text{ cm}$$

Now, $\frac{d^2A}{dx^2} = \frac{-36000000}{x^3}$

$\therefore$ At $x = 200\sqrt{3}$ cm, $\frac{d^2A}{dx^2} < 0$

$\therefore$ Area is maximum at $x = 200\sqrt{3}$ cm and $y = 300\sqrt{3}$ cm

$\therefore \quad y = 3\sqrt{3}$ m and $x = 2\sqrt{3}$ m

Question153

The equation of the normal to the curve $3x^2 - y^2 = 8$, which is parallel to the line $x + 3y = 10$, is MHT CET 2023 (12 May Shift 2)

Options:

A. $x + 3y + 6 = 0$

B. $x + 3y - 3 = 0$

C. $x + 3y + 8 = 0$

D. $x + 3y - 4 = 0$

Answer: C

Solution:

$3x^2 - y^2 = 8$

Differentiating w.r.t. x, we get

$$6x - 2y\frac{dy}{dx} = 0$$
$$\therefore \frac{dy}{dx} = \frac{3x}{y}$$

$\therefore$ Slope of the tangent to the curve is $\frac{3x}{y}$.

$\therefore$ Slope of the normal is $\frac{-y}{3x}$.

It is parallel to line $x + 3y = 10 \Rightarrow$ slope $= -\frac{1}{3}$

$\therefore \frac{-y}{3x} = \frac{-1}{3} \Rightarrow x = y$

$\therefore$ When $x = y$, equation of the curve becomes

$\therefore 3x^2 - x^2 = 8$

$\therefore x^2 = 4$

$\therefore x = 2, -2 \Rightarrow y = 2, -2$

$\therefore (2, 2)$ and $(-2, -2)$ are the points of contact of the normal and the curve.

$\therefore$ Equations are $(y - 2) = \frac{-1}{3}(x - 2)$ or

$$(y + 2) = \frac{-1}{3}(x + 2)$$
$$\text{i.e., } x + 3y - 8 = 0 \text{ or } x + 3y + 8 = 0$$

Question154

If the curves $y^2 = 6x$ and $9x^2 + by^2 = 16$ intersect each other at right angle, then value of " b ' is MHT CET 2023 (12 May Shift 2)

Options:

A. $\frac{9}{2}$

B. 6

C. $\frac{7}{2}$

D. 4

Answer: A

Solution:

$$
\begin{aligned}
& y^2 = 6x \\
& \Rightarrow 2y\frac{dy}{dx} = 6 \Rightarrow \frac{dy}{dx} = \frac{3}{y} \\
& \text{Also, } 9x^2 + by^2 = 16 \\
& \Rightarrow 18x + 2\,by\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-9x}{by}
\end{aligned}
$$

As given curves intersect each other at right angle, their tangents also intersect at right angles.

$$
\begin{aligned}
& \tfrac{3}{y} \times \tfrac{-9x}{by} = -1 \\
\Rightarrow\quad & by^2 = 27x \\
\therefore\quad & \text{(i)} \Rightarrow b(6x) = 27x \\
& \Rightarrow b = \tfrac{9}{2}
\end{aligned}
$$

Question155

A tank with a rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 4 meter and volume is 36 cubic meters. If building of the tank costs ₹100 per square meter for the base and ₹50 per square meter for the sides, then the cost of least expensive tank is MHT CET 2023 (12 May Shift 2)

Options:

A. ₹3000

B. ₹3300

C. ₹2400

D. ₹3500

Answer: B

Solution:

Let length and breadth of the tank be ' x ' m and ' y ' m respectively.

Height of the tank is $4 \mathrm{~m}$.

Volume $= 36 \mathrm{~m}^3$

$$\begin{aligned} & \therefore \quad 4xy = 36 \\ & \therefore \quad xy = 9 \\ & \therefore \quad y = \frac{9}{x} \end{aligned}$$

$\therefore$ Total area of the tank including sides and base $= xy + 2(4x) + 2(4y)$

....[From (i) and (ii)]

$\therefore \quad \mathrm{f}(x) = 9 + 8x + 8\left(\frac{9}{x}\right)$

$$= 9 + 8x + \frac{72}{x}$$

$$\therefore \quad \mathrm{f}'(x) = 8 - \frac{72}{x^2}$$

$\therefore \quad \mathrm{f}'(x) = 0 \Rightarrow x = 3$

$$\Rightarrow y = 3$$

$\therefore$ Required cost $= 100 \times (3 \times 3) + 50$

$$\begin{aligned} & \times (2 \times 4 \times 3 + 2 \times 4 \times 3) \\ & = 900 + 2400 \\ & = ₹3300 \end{aligned}$$

Question156

Water flows from the base of rectangular tank, of depth 16 meters. The rate of flow of the water is proportional to the square root of depth at any time t. If depth is $4 \mathrm{~m}$ when $\mathrm{t} = 2$ hours, then after 3.5 hours the depth (in meters) is MHT CET 2023 (12 May Shift 2)

Options:

A. 0

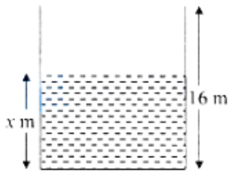
B. 0.25

C. 0.5

D. 3

Answer: B

Solution:



Given that $\frac{dx}{dt} \propto \sqrt{x}$

$\therefore \quad \frac{dx}{dt} = a\sqrt{x}$, for real number a

$\therefore \quad \int \frac{dx}{\sqrt{x}} = \int adt$

$\therefore \quad 2\sqrt{x} = at + c$

When $t = 0, x = 16$

$\therefore \quad$ (i) $\Rightarrow c = 8$

$\therefore \quad$ (i) becomes $2\sqrt{x} =$ at +8

When $t = 2, x = 4$

$\therefore \quad$ (ii) $\Rightarrow a = -2$

$\therefore \quad$ (ii) becomes $2\sqrt{x} = -2t + 8$

$\therefore \quad$ when $t = 3.5$

(iii) $\Rightarrow x = 0.25$ m

Question157

The angle between the tangents to the curves $y = 2x^2$ and $x = 2y^2$ at $(1, 1)$ is MHT CET 2023 (12 May Shift 1)

Options:

A. $\tan^{-1}\left(\frac{15}{8}\right)$

B. $\tan^{-1}\left(\frac{7}{8}\right)$

C. $\tan^{-1}\left(\frac{3}{4}\right)$

D. $\tan^{-1}\left(\frac{1}{4}\right)$

Answer: A

Solution:

$$y=2x^2$$

$\therefore$ Slope of the tangent to this curve is $\frac{dy}{dx}=m_1=4x$

$\therefore$ at $(1,1), m_1=4$

$$x=2y^2$$

$\therefore$ Slope of the tangent to this curve is $\frac{dy}{dx}=m_2=\frac{1}{4y}$

$\therefore$ at $(1,1), m_2=\frac{1}{4}$

Let θ be the angle between two tangents.

$$\begin{aligned} \therefore\quad & \tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|=\left|\frac{4-\frac{1}{4}}{1+4\times\frac{1}{4}}\right|=\frac{15}{8} \\ \therefore\quad & \theta=\tan^{-1}\left(\frac{15}{8}\right)\end{aligned}$$

Question158

If $x=-1$ and $x=2$ are extreme points of $f(x)=\alpha\log x+\beta x^2+x$, α and β are constants, then the value of $\alpha^2+2\beta$ is MHT CET 2023 (12 May Shift 1)

Options:

A. -3

B. 3

C. $\frac{3}{2}$

D. 5

Answer: B

Solution:

According to the given condition, $f'(1)=0$ and $f'(2)=0$

$$f(x)=\alpha\log x+\beta x^2+x$$

$$\begin{aligned} \therefore\quad & f'(x)=\frac{\alpha}{x}+2\beta x+1 \\ \therefore\quad & f'(-1)=0\Rightarrow\alpha+2\beta=1 \\ & \text{and } f'(2)=0\Rightarrow\alpha+8\beta=-2\end{aligned}$$

$\therefore$ From (i) and (ii), we get

$$\beta = \frac{-1}{2} \text{ and } \alpha = 2$$
$$\therefore \quad \alpha^2 + 2\beta = 4 - 1 = 3$$

Question159

The function $\mathrm{f}(x) = \sin^4 x + \cos^4 x$ is increasing in MHT CET 2023 (12 May Shift 1)

Options:

A. $0 < x < \frac{\pi}{8}$

B. $\frac{\pi}{4} < x < \frac{\pi}{2}$

C. $\frac{3\pi}{8} < x < \frac{5\pi}{8}$

D. $\frac{5\pi}{8} < x < \frac{3\pi}{4}$

Answer: B

Solution:

$$\begin{aligned} \therefore \quad \mathrm{f}(x) &= \sin^4 x + \cos^4 x \\ \therefore \quad \mathrm{f}'(x) &= 4\sin^3 x \cos x - 4\cos^3 x \sin x \\ &= 4\sin x \cos x \left(\sin^2 x - \cos^2 x\right) \\ &= +2\sin 2x \cos 2x \\ &= -\sin 4x \end{aligned}$$

$\therefore \quad$ If $\mathrm{f}(x)$ is increasing, then $\mathrm{f}'(x) > 0$

$$\text{i.e.,} \quad \begin{aligned} -\sin 4x > 0 &\Rightarrow \pi < 4x < 2\pi \\ &\Rightarrow \frac{\pi}{4} < x < \frac{\pi}{2} \end{aligned}$$

Question160

In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y. If $x^2 - \mathrm{c}^2 - y$, where c is the length of the third side of the triangle, then the circumradius of the triangle is MHT CET 2023 (12 May Shift 1)

Options:

A. $\frac{c}{3}$

B. $\frac{c}{\sqrt{3}}$

C. $\frac{3}{2}y$

D. $\frac{y}{\sqrt{3}}$

Answer: B

Solution:

Let a and b be the lengths of two sides of a triangle.∴ According to the given condition,

$$\begin{aligned} & \mathrm{a}+\mathrm{b}=x \text{ and } \mathrm{ab}=y \\ & \therefore \quad x^2-\mathrm{c}^2=y \Rightarrow(\mathrm{a}+\mathrm{b})^2-\mathrm{c}^2=\mathrm{ab} \\ & \Rightarrow a^2+b^2+2ab-c^2=ab \\ & \Rightarrow a^2+b^2-c^2=-ab \\ & \Rightarrow \frac{\mathrm{a}^2+\mathrm{b}^2-\mathrm{c}^2}{2\mathrm{ab}}=-\frac{1}{2} \\ & \Rightarrow \cos \mathrm{C}=\frac{1}{2} \\ & \Rightarrow \mathrm{C}=\frac{2\pi}{3} \\ & \Rightarrow \text{ circumradius }=\frac{\mathrm{c}}{2\sin \mathrm{C}}=\frac{\mathrm{c}}{2\sin\left(\frac{2\pi}{3}\right)}=\frac{\mathrm{c}}{\sqrt{3}} \end{aligned}$$

Question161

If the function f is given by $\mathrm{f}(x)=x^3-3(\mathrm{a}-2)x^2+3\mathrm{a}x+7$, for some $\mathrm{a}\in\mathbb{R}$, is increasing in $(0,1]$ and decreasing in $[1,5)$, then a root of the equation $\frac{\mathrm{f}(x)-14}{(x-1)^2}=0(x\neq 1)$ is MHT CET 2023 (11 May Shift 2)

Options:

A. -7

B. 6

C. 7

D. 5

Answer: C

Solution:

$f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$

As $f(x)$ is increasing in $(0, 1]$ and decreasing in $[1, 5)$, we get that $f(x)$ has critical point at $x = 1$
$\Rightarrow f'(1) = 0$

$f'(x) = 3x^2 - 6(a-2)x + 3a$

$\therefore \quad 3(1)^2 - 6(a-2) + 3a = 0$
$\therefore \quad a = 5$
$\therefore \quad \frac{f(x) - 14}{(x-1)^2} = \frac{x^3 - 9x^2 + 15x - 7}{(x-1)^2}$
$= \frac{(x-1)^2(x-7)}{(x-1)^2}$
$= x - 7$

$\therefore$ The required root is 7 .

Question162

The slope of the tangent to a curve $y = f(x)$ at $(x, f(x))$ is $2x + 1$. If the curve passes through the point $(1, 2)$, then the area (in sq. units), bounded by the curve, the X-axis and the line $x = 1$, is MHT CET 2023 (11 May Shift 2)

Options:

A. $\frac{3}{2}$

B. $\frac{4}{3}$

C. $\frac{5}{6}$

D. $\frac{1}{12}$

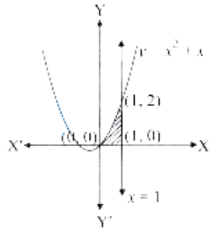
Answer: C

Solution:

According to the given condition, $\frac{dy}{dx} = 2x + 1$ Integrating w.r.t. x, we get $y = x^2 + x + c$

As curve passes through $(1, 2)$, we get $2 = (1)^2 + 1 + c \Rightarrow c = 0$

$\therefore$ The equation of the curve is $y = x^2 + x$.



$\therefore$ Required area

$= \int_0^1 (x^2 + x)\,dx$
$= \left[\frac{x^3}{3} + \frac{x^2}{2}\right]_0^1$
$= \frac{1}{3} + \frac{1}{2}$
$= \frac{5}{6}$ sq. units

Question163

The equation of the tangent to the curve $y = \sqrt{9 - 2x^2}$, at the point where the ordinate and abscissa are equal, is MHT CET 2023 (11 May Shift 2)

Options:

A. $2x + y + \sqrt{3} = 0$

B. $2x + y + 3\sqrt{3} = 0$

C. $2x - y - 3\sqrt{3} = 0$

D. $2x + y - 3\sqrt{3} = 0$

Answer: D

Solution:

Given curve is $y = \sqrt{9 - 2x^2}$

If ordinate and abscissa are equal, we get $y = x$.

$\therefore$ Equation of the curve becomes $x^2 = 9 - 2x^2 \Rightarrow x = \pm\sqrt{3}$

If $x = -\sqrt{3}$, then $y = \sqrt{9 - 2(3)} = \sqrt{3}$

In this case, $x \neq y$.

Hence, $x \neq -\sqrt{3}$

$\therefore$ $x = \sqrt{3}$ and $y = \sqrt{3}$

$\therefore$ Slope of the tangent to the given curve is

$$2y\frac{dy}{dx} = -4x$$

$\therefore$ at $(\sqrt{3}, \sqrt{3})$, $\frac{dy}{dx} = -2$

$\therefore$ Equation of the required tangent is

$$(y - \sqrt{3}) = -2(x - \sqrt{3})$$
i.e., $2x + y - 3\sqrt{3} = 0$

Question164

Value of c satisfying the conditions and conclusions of Rolle's theorem for the function $f(x) = x\sqrt{x + 6}, x \in [-6, 0]$ is MHT CET 2023 (11 May Shift 1)

Options:

A. -4

B. 4

C. 3

D. -3

Answer: A

Solution:

$$\begin{aligned} \mathrm{f}(x) &= x\sqrt{x+6} \\ \therefore \quad \mathrm{f}'(x) &= x\left(\frac{1}{2\sqrt{x+6}}\right)+\sqrt{x+6}(1) \\ &= \frac{x}{2\sqrt{x+6}}+\sqrt{x+6} \end{aligned}$$

Since $\mathrm{f}(x)$ satisfies all the conditions of Rolle's Theorem,There exists $c \in (-6,0)$ such that

$$\begin{aligned} & \mathrm{f}'(\mathrm{c})=0 \\ & \Rightarrow \frac{\mathrm{c}}{2\sqrt{\mathrm{c}+6}}+\sqrt{\mathrm{c}+6}=0 \\ & \Rightarrow \mathrm{c}+2\mathrm{c}+12=0 \\ & \Rightarrow \mathrm{c}=-4 \end{aligned}$$

Question165

If $\mathrm{f}(x)=x\mathrm{e}^{x(1-x)}, x \in \mathrm{R}$, then $\mathrm{f}(x)$ is MHT CET 2023 (11 May Shift 1)

Options:

A. increasing in $\left[-\frac{1}{2}, 1\right]$

B. decreasing R

C. increasing in R

D. decreasing in $\left[-\frac{1}{2}, 1\right]$

Answer: A

Solution:

$$\begin{aligned} \mathrm{f}(x) &= x\mathrm{e}^{x(1-x)} \\ \therefore \quad \mathrm{f}'(x) &= x\mathrm{e}^{x(1-x)}[x(-1)+(1-x)]+\mathrm{e}^{x(1-x)} \\ &= \mathrm{e}^{x(1-x)}\left(x-2x^2+1\right) \end{aligned}$$

For $\mathrm{f}(x)$ to be increasing, $\mathrm{f}'(x) \geq 0$

$$\Rightarrow e^{x(1-x)}\left(x-2x^2+1\right) \geq 0$$
$$\Rightarrow x-2x^2+1 \geq 0$$
$$\Rightarrow 2x^2-x-1 \leq 0$$
$$\Rightarrow (2x+1)(x-1) \leq 0$$
$$\Rightarrow x \in \left[-\frac{1}{2}, 1\right]$$

For $f(x)$ to be decreasing, $f'(x) \leq 0$

$$\Rightarrow (2x+1)(x-1) \geq 0$$
$$\Rightarrow x \in \left(-\infty, -\frac{1}{2}\right] \cup [1, \infty)$$

Question166

A curve passes through the point $\left(1, \frac{\pi}{6}\right)$. Let the slope of the curve at each point (x, y) be $\frac{y}{x}+\sec\left(\frac{y}{x}\right), x>0$, then, the equation of the curve is MHT CET 2023 (11 May Shift 1)

Options:

A. $\sin\left(\frac{y}{x}\right)=\log(x)+\frac{1}{2}$

B. $\operatorname{cosec}\left(\frac{y}{x}\right)=\log(x)+2$

C. $\sec\left(\frac{2y}{x}\right)=\log(x)+2$

D. $\cos\left(\frac{2y}{x}\right)=\log(x)+\frac{1}{2}$

Answer: A

Solution:

$$\frac{dy}{dx}=\frac{y}{x}+\sec\left(\frac{y}{x}\right)$$
$$\text{Put } y=vx$$
$$\therefore \quad \frac{dy}{dx}=v+x\frac{dv}{dx}$$
$$\therefore \quad v+x\frac{dv}{dx}=v+\sec v$$
$$\Rightarrow \cos v dv=\frac{1}{x}\,dx$$

Integrating both sides, we get

$$\sin V=\log(x)+c$$
$$\Rightarrow \sin\left(\frac{y}{x}\right)=\log(x)+c$$

The curve passes through $\left(1, \frac{\pi}{6}\right)$.

$$\sin\left(\frac{\pi}{6}\right) = \log(1) + c \Rightarrow c = \frac{1}{2}$$

$\therefore$ Equation (ii) becomes

$$\sin\left(\frac{y}{x}\right) = \log(x) + \frac{1}{2}$$

Question167

If the surface area of a spherical balloon of radius $6\ \mathrm{cm}$ is increasing at the rate $2\ \mathrm{cm}^2/\mathrm{sec}$, then the rate of increase in its volume in $\mathrm{cm}^3/\mathrm{sec}$ is MHT CET 2023 (10 May Shift 2)

Options:

A. 16

B. 6

C. 12

D. 8

Answer: B

Solution:

Surface area, $S = 4\pi r^2$

$$\begin{aligned}
&\therefore \quad \frac{\mathrm{dS}}{\mathrm{dt}} = 8\pi\mathrm{r}\frac{\mathrm{dr}}{\mathrm{dt}} \\
&\Rightarrow 2 = 8\pi\mathrm{r}\frac{\mathrm{dr}}{\mathrm{dt}} \\
&\Rightarrow \frac{\mathrm{dr}}{\mathrm{dt}} = \frac{1}{4\pi\mathrm{r}} \ldots (i) \\
&\text{Volume, } V = \frac{4}{3}\pi r^3 \\
&\therefore \quad \frac{\mathrm{dV}}{\mathrm{dt}} = \frac{4}{3} \times 3\pi\mathrm{r}^2 \times \frac{\mathrm{dr}}{\mathrm{dt}} \\
&= 4\pi r^2 \times \frac{1}{4\pi r} \ldots [From(i)] \\
&= \mathrm{r} \\
&= 6\ \mathrm{cm}^3/\mathrm{sec}
\end{aligned}$$

Question168

An open metallic tank is to be constructed, with a square base and vertical sides, having volume 500 cubic meter. Then the dimensions of the tank, for minimum area of the sheet metal used in its

construction, are MHT CET 2023 (10 May Shift 1)

Options:

A. $5 \mathrm{~m}, 5 \mathrm{~m}, 10 \mathrm{~m}$

B. $10 \mathrm{~m}, 10 \mathrm{~m}, 5 \mathrm{~m}$

C. $2 \mathrm{~m}, 2 \mathrm{~m}, 8 \mathrm{~m}$

D. $15 \mathrm{~m}, 15 \mathrm{~m}, 5 \mathrm{~m}$

Answer: B

Solution:

Let the length, breadth and depth of open tank be x, x and y respectively.

Volume $(\mathrm{V})=x^2 y$

$\therefore \quad 500=x^2 y$... (i)

Total surface area of open tank is given by

$\mathrm{S}=x^2+4 x y$... (ii)

From (i), $y=\frac{500}{x^2}$

From (ii), $\mathrm{S}=x^2+4 x \times \frac{500}{x^2}$

$=x^2+\frac{2000}{x}$

Differentiating w.r.t. x, we get

$\frac{\mathrm{dS}}{\mathrm{d} x}=2 x-\frac{2000}{x^2}$

For minimum area, $\frac{\mathrm{dS}}{\mathrm{d} x}=0$

$$\begin{aligned} \therefore \quad & 2 x-\frac{2000}{x^2}=0 \\ & \Rightarrow 2000=2 x^3 \\ & \Rightarrow x^3=1000 \end{aligned}$$

$$\begin{aligned} & \Rightarrow x=10 \mathrm{~m} \\ & \frac{\mathrm{d}^2 \mathrm{~S}}{\mathrm{~d} x^2}=2+\frac{4000}{x^3} \\ & \Rightarrow\left(\frac{\mathrm{d}^2 \mathrm{~S}}{\mathrm{~d} x^2}\right)_{x=10}>0 \end{aligned}$$

S is minimum when x = 10 m and y = 5 m

...[From (i)]

Question169

A square plate is contracting at the uniform rate $4 \mathrm{~cm}^2 / \mathrm{sec}$, then the rate at which the perimeter is decreasing, when side of the square is $20 \mathrm{~cm}$, is MHT CET 2023 (10 May Shift 1)

Options:

A. $\frac{1}{5}$ cm/sec.

B. 4 cm/sec.

C. 2 cm/sec.

D. $\frac{2}{5}$ cm/sec.

Answer: D

Solution:

Let A, P and X be the area, perimeter and length of side of square respectively at time ' t ' seconds.
Then,

$$A = X^2, P = 4X$$
$$\therefore \quad P = 4\sqrt{A}$$

Differentiating w.r.t. t, we get

$$\begin{aligned}\frac{dP}{dt} &= 4\frac{1}{2\sqrt{A}} \cdot \frac{dA}{dt} \\ &= \frac{2}{X} \cdot \frac{dA}{dt} \\ &= \frac{2}{20} \times 4 \ldots \begin{bmatrix} \text{side } = 20 \text{ cm} \\ \frac{dA}{dt} = 4 \text{ cm}^2/\text{sec} \end{bmatrix} \\ &= \frac{2}{5} \text{ cm/sec}\end{aligned}$$

Question170

A ladder of length 17 m rests with one end against a vertical wall and the other on the level ground. If the lower end slips away at the rate of 1 m/sec., then when it is 8 m away from the wall, its upper end is coming down at the rate of MHT CET 2023 (10 May Shift 1)

Options:

A. $\frac{5}{8}$ m/sec.

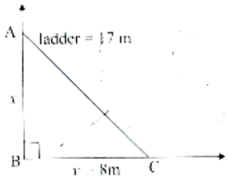
B. $\frac{8}{15}$ m/sec.

C. $\frac{-8}{15}$ m/sec.

D. $\frac{15}{8}$ m/sec.

Answer: B

Solution:



In $\triangle ABC$, AC represents ladder

$AB \rightarrow$ vertical wall

Let $AB = x, BC = y$

$\therefore \quad \angle ABC = 90^{\circ}$

By Pythagoras theorem,

$$AB^2 + BC^2 = AC^2$$
$$\Rightarrow x^2 + y^2 = 17^2$$
$$\Rightarrow x^2 = 289 - y^2 \ldots (i)$$
$$\Rightarrow x^2 = 289 - 64$$
$$\Rightarrow x^2 = 225$$
$$\Rightarrow x = 15 \text{ m}$$

Consider equation (i),

$$x^2 = 289 - y^2$$

Differentiating w.r.t. t, we get

$$2x \frac{dx}{dt} = -2y \frac{dy}{dt}$$
$$\Rightarrow 15 \frac{dx}{dt} = -8(1)$$
$$\Rightarrow \frac{dx}{dt} = \frac{-8}{15} \text{ m/s}$$

Negative sign shows that the ladder is moving down. i.e., vertical length is decreasing

$\therefore \quad$ Upper end is coming down at the rate of $\frac{8}{15}$ m/s.

Question171

A kite is 120 m high and 130 m of string is out. If the kite is moving away horizontally at the rate of 39 m/sec, then the rate at which the string is being out, is MHT CET 2023 (10 May Shift 1)

Options:

A. 12 m/sec.

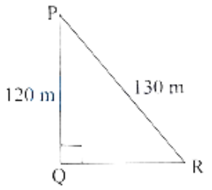
B. 15 m/sec.

C. 18 m/sec.

D. 20 m/sec.

Answer: B

Solution:



Let 'P' be the position of the kite and PR be the string.

Let $QR = x$ and $PR = y$

By Pythagoras theorem,

$$-PR^2 = PQ^2 + QR^2$$
$$\Rightarrow y^2 = (120)^2 + x^2 \ldots (i)$$

Differentiating w.r.t.t, we get

$$2y\frac{dy}{dt} = 2x\frac{dx}{dt}$$
$$\Rightarrow y\frac{dy}{dt} = x\frac{dx}{dt} \ldots (ii)$$

Now, kite is moving away horizontally at the rate of $39\ m/sec$.

$$\therefore \quad \frac{dx}{dt} = 39\ m/sec$$

From (i),

$$(130)^2 = (120)^2 + x^2$$
$$\Rightarrow x^2 = 16900 - 14400$$
$$\Rightarrow x^2 = 2500$$
$$\Rightarrow x = 50$$

From (ii),

$$130\frac{dy}{dt} = 50 \times 39$$
$$\therefore \quad \frac{dy}{dt} = \frac{50 \times 39}{130} = 15\ m/sec$$

Question172

If slope of a tangent to the curve $xy + ax + by = 0$ at the point $(1, 1)$ on it is 2 , then $a - b$ is MHT CET 2023 (09 May Shift 2)

Options:

A. 3

B. 1

C. 2

D. -1

Answer: A

Solution:

Given curve is

$$xy + ax + by = 0$$
$$\therefore \quad \text{Slope } = 2 = \frac{dy}{dx}$$
$$\therefore \quad xy + ax + by = 0$$

Differentiating w.r.t. x, we get

$$x\frac{dy}{dx} + y + a + b\frac{dy}{dx} = 0$$
$$\therefore \quad (x+b)\frac{dy}{dx} = -(y+a)$$
$$\frac{dy}{dx} = \frac{-(y+a)}{x+b}$$
$$\therefore \quad \left(\frac{dy}{dx}\right)_{(1,1)} = 2$$
$$\therefore \quad 2 = \frac{-(1+a)}{1+b}$$
$$\therefore \quad a + 2b = -3 \ldots (i)$$

Since $(1,1)$ lies on $xy + ax + by = 0$, we get $\mathbf{a} + \mathbf{b} = -1$ ... (ii)

Solving (i), (ii), we get

$$a = 1, b = -2$$
$$\therefore \quad a - b = 1 - (-2) = 3$$

Question173

The equation $x^3 + x - 1 = 0$ has MHT CET 2023 (09 May Shift 2)

Options:

A. no real root.

B. exactly two real roots.

C. exactly one real root.

D. all three real roots.

Answer: C

Solution:

Given equation $x^3 + x - 1 = 0$

Let $\mathrm{f}(x) = x^3 + x - 1$

$$\therefore \quad \mathrm{f}'(x) = 3x^2 + 1$$
$$\Rightarrow \mathrm{f}'(x) > 0 \forall x \in \mathrm{R}$$

$\Rightarrow \mathrm{f}(x)$ is increasing

$\therefore$ for $x_2 > x_1, \mathrm{f}(x_2) > \mathrm{f}(x_1)$

Now, $\mathrm{f}(0) = -1$ and $\mathrm{f}(1) = 1$

$\therefore$ $\mathrm{f}(x) = 0$ for some $x \in (0, 1)$

$\therefore$ Equation has one real root.

Question174

The range of values of x for which $f(x) = x^3 + 6x^2 - 36x + 7$ is increasing in MHT CET 2023 (09 May Shift 2)

Options:

A. $(-\infty, -6) \cup (2, \infty)$

B. $(-6, 2)$

C. $(-\infty, -2) \cup (6, \infty)$

D. $(-6, 2]$

Answer: A

Solution:

$$\mathrm{f}(x) = x^3 + 6x^2 - 36x + 7$$
$$\mathrm{f}'(x) = 3x^2 + 12x - 36$$
$$= 3\left(x^2 + 4x - 12\right)$$

For $\mathrm{f}(x)$ to be increasing,

$$\mathrm{f}'(x) > 0$$
$$\Rightarrow 3\left(x^2 + 4x - 12\right) > 0$$
$$\Rightarrow x^2 + 4x - 12 > 0$$
$$\Rightarrow (x + 6)(x - 2) > 0$$
$$\Rightarrow x \in (-\infty, -6) \cup (2, \infty)$$

Question175

The maximum value of the function $f(x) = 3x^3 - 18x^2 + 27x - 40$ on the set $\mathrm{S} = \left\{x \in \mathbb{R} / x^2 + 30 \leq 11x\right\}$ is MHT CET 2023 (09 May Shift 2)

Options:

A. 122

B. 132

C. 112

D. 222

Answer: A

Solution:

$$\begin{aligned}
&S=\{x\in\mathbb{R}/x^2+30\le 11x\}\\
&x^2+30\le 11x\\
&x^2-11x+30\le 0\\
&(x-5)(x-6)\le 0\\
&x\in[5,6]
\end{aligned}$$

Now, $f(x)=3x^3-18x^2+27x-40$

$$\begin{aligned}
f'(x)&=9x^2-36x+27\\
f'(x)&=9\left(x^2-4x+3\right)\\
&=9\left[\left(x^2-4x+4\right)-1\right]\\
&=9(x-2)^2-9
\end{aligned}$$

$\therefore\quad f'(x)>0\forall x\in[5,6]$

$\therefore f(x)$ is strictly increasing in the interval $[5,6]$

$\therefore\quad$ Maximum value of $f(x)$ when $x\in[5,6]$ is

$f(6)=122$

Question176

Let $f'(0)=-3$ and $f'(x)\le 5$ for all real values of x. The $f(2)$ can have possible maximum value as MHT CET 2023 (09 May Shift 2)

Options:

A. 10

B. 5

C. 7

D. 13

Answer: C

Solution:

Applying Lagrange's mean value theorem on interval $[0,2]$, we get there exist atleast one ' c ' $\in(0,2)$ such that

$$\frac{f(2)-f(0)}{2-0}=f'(c)$$

$\therefore\quad f(2)-f(0)=2f'(c)$
$\therefore\quad f(2)=f(0)+2f'(c)$
$\therefore\quad f(2)=-3+2f'(c)$

Given that $f'(x)\leq 5$ for all x

$\therefore\quad f(2)\leq -3+10$
$\therefore\quad f(2)\leq 7$

$\therefore$ Largest possible value of $f(2)$ is 7 .

Question177

The value of c of Lagrange's mean value theorem for $f(x)=\sqrt{25-x^2}$ on $[1,5]$ is MHT CET 2023 (09 May Shift 1)

Options:

A. $\sqrt{15}$

B. 5

C. $\sqrt{10}$

D. 1

Answer: A

Solution:

$$f(x)=\sqrt{25-x^2}$$

$$\therefore\quad f'(x=c)=\frac{-2c}{2\sqrt{25-c^2}}$$

$$=\frac{-c}{\sqrt{25-c^2}}$$

Applying Lagrange's mean value theorem, we get

$$f'(c)=\frac{f(1)-f(5)}{1-5}$$

$$\therefore\quad \frac{c}{\sqrt{25-c^2}}=\frac{\sqrt{25-1}-\sqrt{25-5^2}}{1-5}$$

$$\therefore\quad \frac{-c}{\sqrt{25-c^2}}=\frac{-\sqrt{24}}{4}$$

$\therefore\quad 4c=\sqrt{24}\cdot\sqrt{25-c^2}$
$\therefore\quad 16c^2=24\left(25-c^2\right)$
$\therefore\quad c^2=15$
$\therefore\quad c=\pm\sqrt{15}$

Since $c=-\sqrt{15}$ does not lie in $[1,5]$ $c=\sqrt{15}$

Question178

The approximate value of $\log_{10} 998$ is (given that $\log_{10} \mathrm{e} = 0.4343$) MHT CET 2023 (09 May Shift 1)

Options:

A. 3.0008686

B. 1.9991314

C. 2.0008686

D. 2.9991314

Answer: D

Solution:

Let

$$\begin{aligned} \mathrm{f}(x) = \log_{10} x = \frac{\log_e x}{\log_e 10} &= (\log_{10} \mathrm{e})(\log_e x) \\ &= 0.4343(\log_e x) \end{aligned}$$

On differentiating w.r.t. x, we get

$$\begin{aligned} &\mathrm{f}'(x) = \frac{0.4343}{x} \\ &\text{Let } x = 998 \\ &\quad = 1000 - 2 = \mathrm{a} + \mathrm{h} \\ \therefore \quad &\mathrm{a} = 1000,\ \mathrm{h} = -2 \\ &\mathrm{f}(\mathrm{a}) = \mathrm{f}(1000) \\ &\quad = \log_{10}(1000) \\ &\quad = 3\log_{10} 10 \\ \therefore \quad &\mathrm{f}(\mathrm{a}) = 3 \end{aligned}$$

$$\text{Also, } \mathrm{f}'(\mathrm{a}) = \mathrm{f}'(1000) = \frac{0.4343}{1000} = 0.0004343$$

$$\mathrm{f}(\mathrm{a}+\mathrm{h}) \approx \mathrm{f}(\mathrm{a}) + \mathrm{h}\mathrm{f}'(\mathrm{a})$$

$$\therefore \quad \log_{10}(998) \approx 3 - 2(0.0004343)$$

Question179

The maximum value of xy when $x + 2y = 8$ is MHT CET 2023 (09 May Shift 1)

Options:

A. 20

B. 16

C. 24

D. 8

Answer: D

Solution:

$$x+2y=8$$
$$\therefore \quad 2y=8-x$$
$$\therefore \quad y=\frac{8-x}{2}$$

Let $f(x)=xy$

$\therefore \quad f(x)=x \cdot \frac{(8-x)}{2}$

Differentiating w.r.t x, we get

$$f'(x)=\frac{(8-x)-x}{2}$$
$$f'(x)=4-x$$

To find critical points,

$$f'(x)=0$$
$$\therefore \quad 4-x=0$$
$$\therefore \quad x=4$$

critical point at $x=4$

$\therefore \quad f(4)=\frac{4(8-4)}{2}=8$

$\therefore$ Maximum value of the given function is 8 .

Question180

An object is moving in the clockwise direction around the unit circle $x^2+y^2=1$. As it passes through the point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, its y-co-ordinate is decreasing at the rate of 3 units per sec. The rate at which the x-co-ordinate changes at this point is MHT CET 2023 (09 May Shift 1)

Options:

A. 2 units /sec

B. $3\sqrt{3}$ units /sec

C. $\sqrt{3}$ units /sec

D. $2\sqrt{3}$ units /sec

Answer: B

Solution:

Given equation is $x^2+y^2=1$ Differentiating w.r.t. t, we get

$$\begin{aligned}&2x\frac{\mathrm{d}x}{\mathrm{dt}}+2y\frac{\mathrm{d}y}{\mathrm{dt}}=0\\&2x\frac{\mathrm{d}x}{\mathrm{dt}}+2y(-3)=0\\&\frac{\mathrm{d}x}{\mathrm{dt}}=\frac{6y}{2x}\\&\frac{\mathrm{d}x}{\mathrm{dt}}=\frac{3y}{x}\\&\left.\frac{\mathrm{d}x}{\mathrm{dt}}\right|_{\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)}=\frac{3\times\frac{\sqrt{3}}{2}}{\frac{1}{2}}\\&=3\sqrt{3}\text{ units/sec}\end{aligned}$$

Question181

The rate of change volume of a sphere with respect to its surface area when the radius is 5 meters is MHT CET 2022 (11 Aug Shift 1)

Options:

A. $\frac{2}{5}$

B. 5

C. $\frac{5}{2}$

D. $\frac{1}{2}$

Answer: C

Solution:

$$\begin{aligned}&V=\frac{4}{3}\pi r^3\\&\Rightarrow\frac{\mathrm{d}v}{\mathrm{d}t}=4\pi r^2\frac{\mathrm{d}r}{\mathrm{d}t}\\&S=4\pi r^2\\&\Rightarrow\frac{\mathrm{d}s}{\mathrm{d}t}=8\pi r\frac{\mathrm{d}r}{\mathrm{d}t}\\&\text{Now }\frac{\mathrm{d}v}{\mathrm{d}s}=\frac{\frac{\mathrm{d}v}{\mathrm{d}t}}{\frac{\mathrm{d}s}{\mathrm{d}t}}=\frac{r}{2}=\frac{5}{2}\end{aligned}$$

Question182

If the curves $y^2=6x, 9x^2+by^2=16$ intersect each other at right angles, the value of b is MHT CET 2022 (11 Aug Shift 1)

Options:

A. 4

B. $\frac{7}{2}$

C. 6

D. $\frac{9}{2}$

Answer: D

Solution:

$\left(\frac{dy}{dx}\right)_{C_1} = \frac{3}{y}$ and $\left(\frac{dy}{dx}\right)_{C_2} = \frac{-9x}{by}$

for orthogonal intersection $\left(\frac{dy}{dx}\right)_{C_1} \times \left(\frac{dy}{dx}\right)_{C_2} = -1$

$$\begin{aligned} &\Rightarrow \frac{3}{y} \times \frac{-9x}{by} = -1 \\ &\Rightarrow 27x = by^2 \\ &\Rightarrow 27x = b \times 6x \left[\because y^2 = 6x\right] \\ &\Rightarrow b = \frac{9}{2} \end{aligned}$$

Question183

The radius of right circular cylinder increase at the rate $0.1\ \mathrm{cm/min}$ and height decreases at the rate of $0 \cdot 2\ \mathrm{cm/min}$ The rate of change of volume of the cylinder in $\mathrm{cm}^3/\mathrm{min}$, when the radius is $2\ \mathrm{cm}$ and height is $3\ \mathrm{cm}$, is MHT CET 2022 (11 Aug Shift 1)

Options:

A. $-2\mathrm{pm}^3/\mathrm{min}$

B. $\frac{-3\pi}{5}\ \mathrm{cm}^3/\mathrm{min}$

C. $\frac{-8\pi}{5}\ \mathrm{cm}^3/\mathrm{min}$

D. $\frac{2\pi}{5}\ \mathrm{cm}^3/\mathrm{min}$

Answer: D

Solution:

$$\begin{aligned} &V = pr^2 h \\ &\Rightarrow \frac{dv}{dt} = \pi\left\{2r \cdot \frac{dr}{dt} \cdot h + r^2 \cdot \frac{dh}{dt}\right\} \\ &= \pi\left\{2 \times 2 \times 0 \cdot 1 \times 3 + 2^2 \times (-0 \cdot 2)\right\} \\ &= \pi\{1 \cdot 2 - 0 \cdot 8\} = 0 \cdot 4\pi = \frac{2\pi}{5}\ \mathrm{cm}^3/\mathrm{min} \end{aligned}$$

Question184

The radius of a cylinder is increasing at the rate $2 \mathrm{~cm} / \mathrm{sec}$ and its height is decreasing at the rate of $3 \mathrm{~cm} / \mathrm{sec}$, then the rate of change of volume, when radius is $3 \mathrm{~cm}$ and the height is $5 \mathrm{~cm}$, is MHT CET 2022 (10 Aug Shift 2)

Options:

A. $44 \pi \mathrm{cm}^3 / \mathrm{sec}$

B. $11 \pi \mathrm{cm}^3 / \mathrm{sec}$

C. $23 \pi \mathrm{cm}^3 / \mathrm{sec}$

D. $33 \pi \mathrm{cm}^3 / \mathrm{sec}$

Answer: D

Solution:

$$
\begin{aligned}
& \Rightarrow \frac{\mathrm{d} v}{\mathrm{~d} t}=\pi\left(2 r \frac{\mathrm{d} r}{\mathrm{~d} t} h+r^2 \frac{\mathrm{d} h}{\mathrm{~d} t}\right) \\
V=\pi r^2 h & \Rightarrow \frac{\mathrm{d} v}{\mathrm{~d} t}=\pi\left(2 \times 3 \times 2 \times 5+3^2 \times(-3)\right) \\
& \Rightarrow \frac{\mathrm{d} v}{\mathrm{~d} t}=33 \pi \mathrm{cm}^3 / \mathrm{sec}
\end{aligned}
$$

Question185

For every value of $x \in[1,3]$, the function $f(x)=\frac{1}{8^x}$ is MHT CET 2022 (10 Aug Shift 2)

Options:

A. increasing for $x>2$ and decreasing for $x \leq 2$.

B. neither increasing nor decreasing.

C. decreasing.

D. increasing.

Answer: C

Solution:

$$
\begin{aligned}
& f(x)=\frac{1}{8^x}=8^{-x} \\
& \Rightarrow f^{\prime}(x)=8^{-x} \log 8 \times(-1) \\
& \Rightarrow f^{\prime}(x)=\frac{-\log 8}{8^x}<0 \forall x \in[1,3] \\
& \Rightarrow f(x) \text { is decreasing } \forall x \in[1,3]
\end{aligned}
$$

Question186

The curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2, n \in N$ touches the line at the point (a, b), then the equation of the line is MHT CET 2022 (10 Aug Shift 2)

Options:

A. $\frac{x}{a}-\frac{y}{b}=2$

B. $\frac{x}{a}+\frac{y}{2b}=1$

C. $\frac{x}{a}+\frac{y}{b}=1$

D. $\frac{x}{a}+\frac{y}{b}=2$

Answer: D

Solution:

Slope of the line $=\frac{dy}{dx}$ at (a, b)

$$=\frac{-n\left(\frac{x}{a}\right)^{n-1}\cdot\frac{1}{a}}{n\cdot\left(\frac{y}{b}\right)^{n-1}\cdot\frac{1}{b}} \text{ at } (a, b)$$
$$=\frac{-b}{a}$$

now equation of the straight line

$$y-b=-\frac{b}{a}(x-a)$$
$$\Rightarrow ay-ab=-bx+ab$$
$$\Rightarrow bx+ay=2ab$$
$$\Rightarrow \frac{x}{a}+\frac{y}{b}=2$$

Question187

The set of all points, for which $f(x)=x^2\cdot e^{-x}$ strictly increases, is MHT CET 2022 (10 Aug Shift 1)

Options:

A. $(0, 2)$

B. $(-\infty, \infty)$

C. $(-2, 0)$

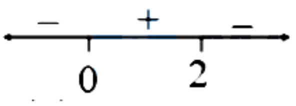
D. $(2, \infty)$

Answer: A

Solution:

$f(x)=x^2e^{-x}$

null



$f(x)$ is strictly increasing in $(0, 2)$

Question188

The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. The rate at which the area increases, when side is 10 cm, is MHT CET 2022 (10 Aug Shift 1)

Options:

A. $\frac{\sqrt{3}}{10} \text{ cm}^2/\text{sec}$

B. $\frac{10}{\sqrt{3}} \text{ cm}^2/\text{sec}$

C. $\sqrt{3} \text{ cm}^2/\text{sec}$

D. $10\sqrt{3} \text{ cm}^2/\text{sec}$

Answer: D

Solution:

$$A = \frac{\sqrt{3}}{4} S^2$$

$$\Rightarrow \frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\sqrt{3}}{4} \cdot 2S \frac{\mathrm{d}s}{\mathrm{d}t} = \frac{\sqrt{3}}{4} \times 2 \times 10 \times 2 \text{ cm}^2/\text{sec}$$

$$\Rightarrow \frac{\mathrm{d}A}{\mathrm{d}t} = 10\sqrt{3} \text{ cm}^2/\text{sec}$$

Question189

A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of $50 \text{ cm}^3/\text{min}$. When the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases, is MHT CET 2022 (10 Aug Shift 1)

Options:

A. $\frac{1}{36\pi} \text{cm/min}$

B. $\frac{5}{6\pi} \text{cm/min}$

C. $\frac{1}{54\pi} \text{cm/min}$

D. $\frac{1}{18\pi} \text{cm/min}$

Answer: D

Solution:

$$V=\frac{4}{3}\pi r^3$$
$$\Rightarrow \frac{\mathrm{d}v}{\mathrm{d}t}=4\pi r^2\frac{\mathrm{d}r}{\mathrm{d}t}$$
$$\Rightarrow 50=4\pi\times 15^2\times\frac{\mathrm{d}r}{\mathrm{d}t}$$
$$\Rightarrow \frac{\mathrm{d}r}{\mathrm{d}t}=\frac{50}{4\pi\times 15^2}=\frac{1}{18\pi}\mathrm{cm/min}$$

Question190

On the interval $[0,1]$, the function $x^{25}(1-x)^{75}$ takes its maximum value at the point MHT CET 2022 (08 Aug Shift 2)

Options:

A. $\frac{1}{2}$

B. 0

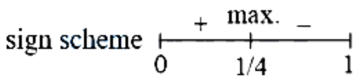
C. $\frac{1}{4}$

D. $\frac{1}{3}$

Answer: C

Solution:

$$f(x)=x^{25}(1-x)^{75}$$
$$\Rightarrow f'(x)=25x^{24}(1-x)^{75}-x^{25}\cdot 75(1-x)^{74}$$
$$=25\cdot x^{24}\cdot(1-x)^{74}(1-4x)$$



$\Rightarrow f(x)$ takes maximum value at $x=\frac{1}{4}$

Question191

Let AD and BC be two vertical poles at A and B respectively on a horizontal ground. If $AD=8\text{ m}$, $BC=11\text{ m}$ and $AB=10\text{ m}$, then the distance (in meters) of point M on AB from the point A such that MD^2+MC^2 is minimum, is MHT CET 2022 (08 Aug Shift 2)

Options:

A. 8

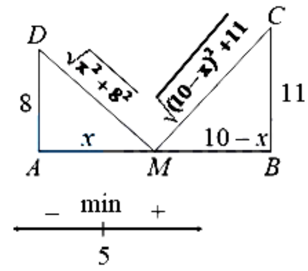
B. 5

C. 4

D. 7

Answer: B

Solution:



$$f(x) = MD^2 + MC^2$$
$$\Rightarrow f(x) = x^2 + 8^2 + (10 - x)^2 + 11^2$$
$$\Rightarrow f'(x) = 2x - 2(10 - x)$$
$$\Rightarrow f'(x) = 4x - 20$$

for $f(x)$ to be minimum $x = 5$

Question192

The tangent to the curve $y = x^3 + ax - b$ at the point $(1, -5)$ is perpendicular to the line $y - x + 4 = 0$, then which one of the following points lies on the curve? MHT CET 2022 (08 Aug Shift 2)

Options:

A. (2,-2)

B. (-2,2)

C. (-2,1)

D. (2,-1)

Answer: A

Solution:

$y = x^3 + ax - b$

slope of tangent $= \frac{dy}{dx} = 3x^2 + a$

slope of the line $y - x + 4 = 0$ is 1

$$\Rightarrow \frac{dy}{dx}_{at(1-5)} = -1$$
$$\Rightarrow 3 \times 1^2 + a = -1$$
$$\Rightarrow a = -4$$

also $(1, -5)$ lies on the curve $y = x^3 + ax - b$

$$\Rightarrow -5 = 1^3 + a \times 1 - b = 1 + (-4) \times 1 - b$$
$$\Rightarrow b = 2$$

Hence, the curve is $y = x^3 - 4x - 2$ which is satisfied by $(2, -2)$

Question193

If the water is being poured at the rate $36 \mathrm{~m}^3/\mathrm{sec}$ in cylindrical vessel of base radius $3 \mathrm{~m}$, then the rate at which water level is rising, is MHT CET 2022 (08 Aug Shift 1)

Options:

A. $\frac{4}{\pi} \mathrm{m}/\mathrm{sec}$

B. $4\pi \mathrm{m}/\mathrm{sec}$

C. $\frac{\pi}{4} \mathrm{~m}/\mathrm{sec}$

D. $\frac{3}{\pi} \mathrm{m}/\mathrm{sec}$

Answer: A

Solution:

$\frac{dv}{dt} = 36 \mathrm{~m}^3/\mathrm{sec}$

and $v = \pi r^2 \mathrm{~h} = \pi \times 3^2 \mathrm{~h} = 9\pi \mathrm{h}$

$\Rightarrow \frac{dv}{dt} = 9\pi \frac{dh}{dt}$

$\Rightarrow 36 = 9\pi \frac{dh}{dt}$

$\Rightarrow \frac{dh}{dt} = \frac{4}{\pi} \mathrm{m}/\mathrm{sec}$

Question194

The function, $f(x) = x\sqrt{1-x}$, where $x \in (0,1)$, has local maximum at $x =$ MHT CET 2022 (08 Aug Shift 1)

Options:

A. $\frac{1}{3}$

B. $\frac{1}{4}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

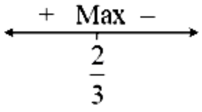
Answer: C

Solution:

$f(x) = x\sqrt{1-x}$

$\Rightarrow f'(x) = 1\sqrt{1-x} + \frac{x}{2\sqrt{1-x}} x(-1) = \frac{2-3x}{2\sqrt{1-x}}$

sign scheme of $f'(x)$



Hence, local maxima at $x = \frac{2}{3}$

Question195

If $x=t^2$ and $y=2t$ are parametric equations of a curve, then equation of the normal to the curve at $t=2$ is MHT CET 2022 (08 Aug Shift 1)

Options:

A. $2x+y-12=0$

B. $x+y-8=0$

C. $x+2y-12=0$

D. $2x+3y-20=0$

Answer: A

Solution:

$$(y-2t)=\frac{-\mathrm{d}x}{\mathrm{d}t}\left(x-t^2\right)$$

$$\begin{aligned}\text{Equation of normal at } t &\Rightarrow (y-2t)=\frac{-2t}{2}\left(x-t^2\right)\\ &\Rightarrow (y-4)=-2(x-4)[\text{ for } t=2]\\ &\Rightarrow 2x+y-12=0\end{aligned}$$

Question196

The rate of disintegration of a radio active element at time t is proportional to its mass at that time. Then the time during which the original mass of $6\mathrm{gm}$ will disintegrate into its mass of $3\mathrm{gm}$ is proportional to MHT CET 2022 (07 Aug Shift 2)

Options:

A. $\log 4$

B. $\log 3$

C. $\log 5$

D. $\log 2$

Answer: D

Solution:

$$\frac{d(m)}{dt} \propto m$$
$$\Rightarrow \frac{dm}{dt} = -km$$
$$\int \frac{dm}{m} = -\int kdt$$
$$\Rightarrow \log_e m = -kt + c$$
$$\Rightarrow m = e^{-kt+c} = e^c \cdot e^{-kt}$$
at $t = 0, m = 6 \Rightarrow e^c = 6$
i.e., $m = 6 \cdot e^{-kt}$

at $t = t, m = 3$

Hence $3 = 6 \cdot e^{-kt}$

$$\Rightarrow \frac{1}{2} = e^{-kt}$$
$$\Rightarrow \log\left(\frac{1}{2}\right) = -kt$$
$$\Rightarrow -\log 2 = -kt$$
$$\Rightarrow t = \frac{\log 2}{k} \text{ [Propotional to } \log 2]$$

Question197

If $f(x) = \frac{x}{\log x}$, then $f(x)$ is increasing in MHT CET 2022 (07 Aug Shift 2)

Options:

A. $(0, \infty)$

B. (e, ∞)

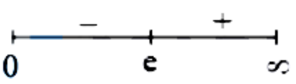
C. $(-\infty, 0)$

D. $[e, \infty)$

Answer: D

Solution:

$$y = \frac{x}{\log x} \Rightarrow \frac{dy}{dx} = \frac{\log x \times 1 - x \times \frac{1}{x}}{(\log x)^2} = \frac{(\log x) - 1}{(\log x)^2}$$

sign scheme 

$f(x)$ is increasing in $[e, \infty)$

Question198

Two positive numbers x and y such that $(x + y) = 60$ and xy^3 is maximum, then the numbers are respectively MHT CET 2022 (07 Aug Shift 2)

Options:

A. 45,15

B. 30,30

C. 20,40

D. 40,20

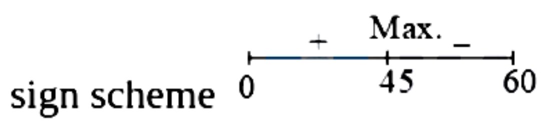
Answer: A

Solution:

$x+y=60 \Rightarrow x=60-y$

Now let $z=xy^3=(60-y)y^3=60y^3-y^4$

$$\Rightarrow \frac{dz}{dy}=180y^2-4y^3=4y^2(45-y)$$



i.e., xy^3 is maximum $y=45$ and $x=60-45=15$

Question199

The function $f(x)=2x^3-9x^2+12x+29$ is monotonically increasing in the interval MHT CET 2022 (07 Aug Shift 1)

Options:

A. $(-\infty,1)\cup(2,\infty)$

B. $(-\infty,\infty)$

C. $(2,\infty)$

D. $(-\infty,1)$

Answer: A

Solution:

$f(x)=2x^3-9x^2+12x+29$

$\Rightarrow f'(x)=6x^2-18x+12=6(x-1)(x-2)$ sign scheme

$\Rightarrow f(x)$ is increasing in the interval

$x\varepsilon(\infty-,1)\cup(2,\infty)$

Question200

20 meters of wire is available to fence of a flowerbed in the form of a circular sector. If the flowerbed is to have maximum surface area, then the radius of the circle is MHT CET 2022 (07 Aug Shift 1)

Options:

A. 8 m

B. 4 m

C. 2 m

D. 5 m

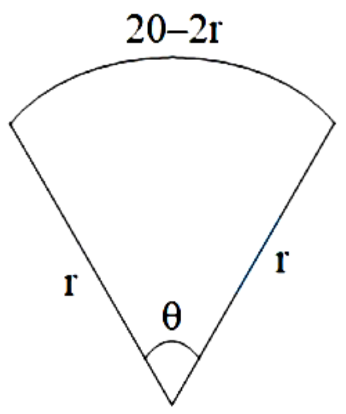
Answer: D

Solution:

$\theta^c = \frac{\ell}{r} = \frac{20-2r}{r} = \frac{20-2r}{r} \times \frac{180^\circ}{\pi}$

Now area of sector $= \frac{\pi r^2 \theta}{360^\circ}$

$A(r) = \frac{\pi r^2 \times \frac{20-2r}{r} \times \frac{180^\circ}{\pi}}{360^\circ} = 10r - r^2$



For maximum area $A'(r) = 0$

$\Rightarrow 10 - 2r = 0$

$\Rightarrow r = 5 \text{ m}$

Question201

If $y = 4x - 5$ is tangent to the curve $y^2 = px^3 + q$ at $(2, 3)$, then MHT CET 2022 (07 Aug Shift 1)

Options:

A. $p = 2, q = -7$

B. $p = 2, q = 7$

C. $p = -2, q = 7$

D. $p = -2, q = -7$

Answer: A

Solution:

$(2,3)$ lies on $y^2 = px^3 + q$
$\Rightarrow 3^2 = P \times 2^3 + q$

$\Rightarrow 8P + q = 9$ ...(i)

Also $2y\frac{dy}{dx} = 3px^2 \Rightarrow \frac{dy}{dx} = \frac{3px^2}{2y}$

$\frac{dy}{dx}$ at $(2,3) = 4 \Rightarrow \frac{3P \times 2^2}{2 \times 3} = 4 \Rightarrow P = 2$ ...(ii)

from (i) and (ii) $q = -7$

Question202

If surrounding air is kept at $20^\circ\mathrm{C}$ and body cools from $80^\circ\mathrm{C}$ to $70^\circ\mathrm{C}$ in 5 minutes, then the temperature of the body after 15 minute will be MHT CET 2022 (07 Aug Shift 1)

Options:

A. $52.7^\circ\mathrm{C}$

B. $51.7^\circ\mathrm{C}$

C. $54.7^\circ\mathrm{C}$

D. $50.7^\circ\mathrm{C}$

Answer: C

Solution:

$\frac{dT}{dt} = -k(T-20) \Rightarrow T - 20 = e^{-kt+c} \Rightarrow T = 20 + e^c e^{-kt}$

for $t = 0, T = 80 \Rightarrow e^c = 60$

i.e. $T = 20 + 60 \cdot e^{-kt}$

For $t = 5, T = 70 \Rightarrow 70 = 20 + 60e \Rightarrow -5k = \log \frac{5}{6}$

i.e. $T = 20 + 60e^{\frac{1}{5}\log\frac{5}{6} \times t}$

Now, for $t = 15$

$T = 20 + 60e^{15 \times \frac{1}{5}\log\frac{5}{6}} = 20 + 60 \times \left(\frac{5}{6}\right)^3 = 54.7$

Question203

A firm is manufacturing 2000 items. It is estimated that the rate of change of production P with respect to additional number of workers x is given by $\frac{dp}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is MHT CET 2022 (06 Aug Shift 2)

Options:

A. 2500

B. 3000

C. 3500

D. 4500

Answer: C

Solution:

$$\frac{dp}{dx}=100-12\sqrt{x}\Rightarrow\int dp=\int(100-12\sqrt{x})dx$$
$$\Rightarrow p=100x-12\times\frac{2}{3}x^{3/2}+C$$
$$\Rightarrow p=100x-8x^{3/2}+200[\because\text{ at }x=0,p=2000]$$
$$\Rightarrow p=100\times 25-8\times(25)^{3/2}+2000[\text{ putting }x=25]$$
$$\Rightarrow p=2500-1000+2000=3500$$

Question204

A spherical iron ball of 10 cm radius is coated with a layer of ice of uniform thickness that melts at the rate of $50\text{ cm}^3/\text{min}$. If the thickness of ice is 5 cm, then the rate at which the thickness of ice decrease is MHT CET 2022 (06 Aug Shift 2)

Options:

A. $\frac{1}{18\pi}\text{cm}/\text{min}$

B. $\frac{2}{9\pi}\text{cm}/\text{min}$

C. $\frac{-1}{18\pi}\text{cm}/\text{min}$

D. $\frac{1}{3\pi}\text{cm}/\text{min}$

Answer: A

Solution:

$$v=\frac{4}{3}\pi r^3$$
$$\frac{dv}{dt}=50\quad r=(10+5)=15\text{ cm}$$
$$\Rightarrow\frac{dv}{dt}=4\pi r^2\frac{dr}{dt}$$
$$\Rightarrow 50=4\pi\times 15\times 15\times\frac{dr}{dt}$$
$$\Rightarrow\frac{dr}{dt}=\frac{50}{4\pi\times 15\times 15}=\frac{1}{18\pi}\text{cm}/\text{min}$$

Question205

If the normal to the curve $y=f(x)$ at the point $(3,4)$ makes and angle $\left(\frac{3\pi}{4}\right)^{\text{C}}$ with positive X-axis, then $f'(3)$ is equal to MHT CET 2022 (06 Aug Shift 2)

Options:

A. -1

B. 1

C. $\frac{4}{3}$

D. $-\frac{3}{4}$

Answer: B

Solution:

$$\text{Slope of normal } = \frac{-1}{f'(x)}$$
$$\Rightarrow \tan\left(\frac{3\pi}{4}\right) = \frac{-1}{f'(3)}$$
$$\Rightarrow -1 = \frac{-1}{f'(3)}$$
$$\Rightarrow f'(3) = 1$$

Question206

Local maximum and local minimum values respectively of the function $f(x) = (x-1)(x+2)^2$ are MHT CET 2022 (06 Aug Shift 1)

Options:

A. $-4, 0$

B. $0, -4$

C. $-4, 4$

D. $4, -4$

Answer: B

Solution:

$$f(x) = (x-1)(x+2)^2$$
$$f'(x) = (x+2)^2 + (x-1)2(x+2) = (x+2)(x+2+2x-2) = 3x(x+2)$$
$$\overset{+}{+} -0^{+}_{-2}$$
$$f_{\max} = f(-2) = 0$$
$$f_{\min} = f(0) = -4$$

Question207

The function $f(x) = \frac{\log(\pi+x)}{\log(e+x)}$ is MHT CET 2022 (06 Aug Shift 1)

Options:

A. decreasing on $\left(0, \frac{\pi}{e}\right)$, increasing on $\left(\frac{\pi}{e}, \infty\right)$

B. increasing on $\left(0, \frac{\pi}{e}\right)$, decreasing on $\left(\frac{\pi}{e}, \infty\right)$

C. increasing on $(0, \infty)$

D. decreasing on $(0, \infty)$

Answer: D

Solution:

$$\begin{aligned}
&\mathrm{f}(\mathrm{x})=\frac{\log (\pi+\mathrm{x})}{\log (\mathrm{e}+\mathrm{x})} \\
&\Rightarrow \mathrm{f}^{\prime}(\mathrm{x})=\frac{\frac{1}{\pi+\mathrm{x}} \log (\mathrm{e}+\mathrm{x})-\log (\pi+\mathrm{x}) \times \frac{1}{\mathrm{e}+\mathrm{x}}}{\{\log (\mathrm{e}+\mathrm{x})\}^2} \\
&\Rightarrow \mathrm{f}^{\prime}(\mathrm{x})=\frac{(\mathrm{e}+\mathrm{x}) \log (\mathrm{e}+\mathrm{x})-(\pi+\mathrm{x}) \log (\pi+\mathrm{x})}{(\pi+\mathrm{x})(\mathrm{e}+\mathrm{x})\{\log (\mathrm{e}+\mathrm{x})\}^2}
\end{aligned}$$

Let $g(x)=(e+x) \log (e+x)-(\pi+x) \log (\pi+x)$

$\Rightarrow \mathrm{g}^{\prime}(\mathrm{x})=\log (\mathrm{e}+\mathrm{x})-\log (\pi+\mathrm{x})<0$

So, $\mathrm{g}(\mathrm{x})$ is decreasing

$$\begin{aligned}
&\Rightarrow \mathrm{g}(\mathrm{x})<\mathrm{g}(0) \forall \mathrm{x} \in(0, \infty) \\
&\Rightarrow(\mathrm{e}+\mathrm{x}) \log (\mathrm{e}+\mathrm{x})-(\pi+\mathrm{x}) \log (\pi+\mathrm{x})<\mathrm{e} \log \mathrm{e}-\pi \log \pi<0 \\
&\Rightarrow \mathrm{f}^{\prime}(\mathrm{x})<0 \forall \mathrm{x} \in(0, \infty) \\
&\Rightarrow \mathrm{f}(\mathrm{x}) \text { is decreasing on }(0, \infty)
\end{aligned}$$

Question208

If the tangent to the curve $y=\frac{x}{x^2-3}, x \in R,(x \neq \pm \sqrt{3})$ at a point $(\alpha, \beta) \neq(0,0)$ on it, is parallel to the line $2x+6y-11=0$, then MHT CET 2022 (06 Aug Shift 1)

Options:

A. $|2\alpha+6\beta|=11$

B. $|6\alpha+2\beta|=9$

C. $|6\alpha+2\beta|=19$

D. $|2\alpha+6\beta|=19$

Answer: C

Solution:

$y=\frac{x}{x^2-3} \Rightarrow \frac{dy}{dx}=\frac{-(3+x^2)}{(x^2-3)^2}$

Now slope of $2x+6y-11=0$ is $\frac{-1}{3}$

$$\frac{\mathrm{A}}{\mathrm{Q}}-\frac{1}{3}=\frac{-\left(3+x^2\right)}{\left(x^2-3\right)^2}$$
$$\Rightarrow x^4-9x^2=0$$
$$\Rightarrow x^2\left(x^2-9\right)=0$$
$$\Rightarrow x=0 \text{ or } x=\pm 3$$

But $x \neq 0$ so $x=\pm 3 \Rightarrow y=\pm \frac{1}{2}$

Hence, $\alpha=\pm 3, \beta=\pm \frac{1}{2}$

$\Rightarrow|6\alpha+2\beta|=\left|\pm\left(6\times 3+2\times \frac{1}{2}\right)\right|=19$

Question209

The side of a square sheet of metal is increasing at the rate of $3 \mathrm{~cm/min}$. At what rate is the area increasing when the length of the side is $6 \mathrm{~cm}$ long? MHT CET 2022 (05 Aug Shift 2)

Options:

A. $36 \mathrm{~cm}^2/\mathrm{min}$

B. $12 \mathrm{~cm}^2/\mathrm{min}$

C. $18 \mathrm{~cm}^2/\mathrm{min}$

D. $9 \mathrm{~cm}^2/\mathrm{min}$

Answer: A

Solution:

$\mathrm{A}=\mathrm{a}^2$ and $\frac{\mathrm{da}}{\mathrm{dt}}=3 \mathrm{~cm/min}$

Now $\frac{\mathrm{dA}}{\mathrm{dt}}=2\mathrm{a}\frac{\mathrm{dA}}{\mathrm{dt}}$

$\Rightarrow \frac{\mathrm{dA}}{\mathrm{dt}}=2\times 6\times 3 \mathrm{~cm}^2/\mathrm{min}=36 \mathrm{~cm}^2/\mathrm{min}$

Question210

A circular sector of perimeter 60 meter with maximum area is to be constructed. The radius of the circular are in meter must be MHT CET 2022 (05 Aug Shift 2)

Options:

A. 5 m

B. 15 m

C. 10 m

D. 20 m

Answer: B

Solution:

Perimeter $= 2r + r\theta = 60$

$\Rightarrow \theta = \frac{60-2r}{r}$

Area $= \mathrm{A(r)} = \frac{\pi r^2 \theta}{360} = \frac{\pi r^2\left(\frac{60-2r}{r}\right)}{360} = \frac{\pi}{360}\left(60r - 2r^2\right)$

$A'(r) = \frac{\pi}{360}(60 - 4r)$

For area to be maximum $A'(r) = 0 \Rightarrow r = 15$

Question211

The rate at which the population of a city increases varies as the population present. Within the period of 30 years, the population grew from 20 lakhs to 40 lakhs. Then, the population after a further period of 15 years will be(Take $\sqrt{2} = 1.41$) MHT CET 2022 (05 Aug Shift 2)

Options:

A. 56 lakhs

B. 60 lakhs

C. 57.4 lakhs

D. 56.4 lakhs

Answer: D

Solution:

$$\frac{dp}{dt} \propto p \Rightarrow \frac{dp}{dt} = kp \Rightarrow \int \frac{dp}{p} = \int kdt$$
$$\Rightarrow \log_e P = kt + C$$

Putting $t = 0$ year and $p = 20$ lakhs

We get $c = \log_e 20$

$\Rightarrow \log_e P = Kt + \log_e 20$

Again putting $t = 30$ years and $P = 40$ lakhs

$$\Rightarrow \log_e 40 = K \times 30 + \log_e 20$$
$$\Rightarrow K = \frac{\log_e 2}{30}$$
$$\Rightarrow \log_e P = \frac{\log_e 2}{30} t \log_e 20$$

Putting $t = 45$ years

We get $\log_e p = \frac{\log_e 2}{30} \times 45 + \log_e 20$

$$\Rightarrow \log_e P = \frac{3}{2} \log_e 2 + \log_e 20$$
$$\Rightarrow \log_e P = \log_e 2 \frac{3}{2} \times 20$$
$$\Rightarrow P = 2\sqrt{2} \times 20 \text{ lakhs}$$

$\Rightarrow P = 2 \times 1.41 \times 20$ lakhs

$\Rightarrow P = 56.4$ lakhs

Question212

The function $f(x)$ is defined by $f(x) = (x+2)e^{-x}$ is MHT CET 2022 (05 Aug Shift 2)

Options:

A. monotonically decreasing in $(-1, \infty)$ and monotonically increasing in $(-\infty, -1)$

B. decreasing for all x

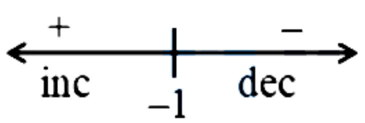
C. increasing for all x

D. decreasing in $(-\infty, -1)$ and increasing in $(-1, \infty)$

Answer: A

Solution:

$$f(x) = (x+2)e^{-x} \Rightarrow fP'(x) = e^{-x} - (x+2)e^{-x} = e^{-x}(1 - x - 2)$$
$$\Rightarrow f'(x) = -e^{-x}(1+x) \text{ sign scheme}$$



Question213

The maximum value of the function $f(x) = 3x^3 - 18x2 + 27x - 40$ on the set $S = \{x \in Rx^2 + 30 \leq 11x\}$ is MHT CET 2022 (05 Aug Shift 1)

Options:

A. 122

B. 222

C. 810

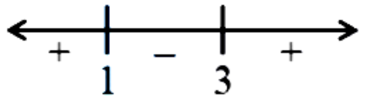
D. 162

Answer: A

Solution:

$f(x) = 3x^3 - 18x^2 + 27x - 40$
$\Rightarrow f'(x) = 9x^2 - 36x + 27$
$= 9(x-1)(x-3)$

Sign scheme for $f'(x)$



Now, $x^2 + 30 \leq 11x$

$\Rightarrow (x-5)(x-6) \leq 0$

$\Rightarrow x \in [5, 6]$

For the interval $[5, 6]$ $f(x)$ is increasing

Hence $f_{\max} = f(6)$

$\forall x \in [5, 6]$ i.e. on the set S

$= 3 \times 6^3 - 18 \times 6^2 + 27 \times 6 - 40 = 122$

Question214

A stone is dropped in a quiet lake and it is observed that waves move in circles, If the radius of a circular wave increases at the rate $2\ \mathrm{cm/sec}$, then the rate of increase in its area at the instant when its radius is $10\ \mathrm{cm}$, is $\mathrm{cm}^2/\mathrm{sec}$. MHT CET 2022 (05 Aug Shift 1)

Options:

A. 40π

B. 80π

C. 10π

D. 20π

Answer: A

Solution:

Given $\frac{dr}{dt}=2\text{ cm/sec} r=10\text{ cm}$

We have $A=\pi r^2$

Diff. w.r.t. t

$$\begin{aligned}\frac{dA}{dt}&=2\pi r\frac{dr}{dt}\\&=2\pi\times 10\times 2\\&=40\pi\end{aligned}$$

Question215

The curve $y=ax^3+bx^2+cx+5$ touches X-axis at $P(-2,0)$ and cuts Y-axis at a point Q, where its gradient is 3 , then MHT CET 2021 (24 Sep Shift 2)

Options:

A. $\mathrm{a}=\frac{1}{2},\ \mathrm{b}=\frac{3}{4},\mathrm{c}=3$

B. $\mathrm{a}=\frac{1}{2},\ \mathrm{b}=\frac{-1}{4},\mathrm{c}=-3$

C. $a=\frac{1}{2},b=\frac{-3}{4},c=-3$

D. $\mathrm{a}=\frac{-1}{2},\ \mathrm{b}=\frac{-3}{4},\mathrm{c}=3$

Answer: D

Solution:

The curve $y=ax^3+bx^2+cx+5$ touches X-axis at $P(-2,0)$

$\therefore 0=\mathrm{a}(-2)^3+\mathrm{b}(-2)^2+\mathrm{c}(-2)+5$

$\therefore$ 8a – 4b + 2c = 5 … (1)

$\frac{dy}{dx}=3ax^2+2bx+c$ and at point Q on Y axis, we have $\frac{dy}{dx}=3$.

Let $Q\equiv(0,k)$

$\therefore$ 3 = 3a(0) + 2(0) + c $\Rightarrow$ c = 3 … (2)

The equation (1) becomes $8a-4b+6=5$ i.e. $8a-4b=-1$

$\Rightarrow 2\mathrm{a}-\mathrm{b}=\frac{-1}{4}$

At $\mathrm{P}(-2,0)$

$\frac{\mathrm{dy}}{\mathrm{dx}}=0$

12a – 4b + c = 0 ….(3)

From (1), (2) \& (3)

$\mathrm{a}=\frac{-1}{2},\ \mathrm{b}=\frac{-3}{4},\mathrm{c}=3$

Question216

The minimum value of the function $f(x)=x\log x$ is MHT CET 2021 (24 Sep Shift 2)

Options:

A. -e

B. e

C. $\frac{1}{e}$

D. $-\frac{1}{e}$

Answer: D

Solution:

$f(x) = x \log x$

$\therefore \quad f'(x) = \frac{x}{x} + \log x = 1 + \log x$

When $1 + \log x = 0 \Rightarrow x = \frac{1}{e}$

$f''(x) = \frac{1}{x} \Rightarrow [f''(x)]_{x=\frac{1}{e}} = e > 0$

Thus $f(x)$ is minimum at $x = \frac{1}{e}$ and

$f(x) = \left(\frac{1}{e}\right) \log\left(\frac{1}{e}\right) = \frac{-1}{e}$

Question217

The distance ' s ' in meters covered by a particle in t seconds is given by $s = 2 + 27t - t^3$. The particle will stop after ——— distance. MHT CET 2021 (24 Sep Shift 2)

Options:

A. 65 meters

B. 80 meters

C. 56 meters

D. 60 meters

Answer: C

Solution:

$s = 2 + 27t - t^3$ and particle stops when its velocity is zero.
$\therefore \frac{ds}{dt} = 27 - 3t^2 = 0 \Rightarrow t^2 = 9 \Rightarrow t = 3sec. \therefore$ Distance covered in 3sec, is
$S_{(t=3)} = 2 + 27(3) - (3)^3 = 56$ m

Question218

The equation of tangent to the curve $y = y = \sqrt{2} \sin\left(2x + \frac{\pi}{4}\right)$ at $x = \frac{\pi}{4}$, is MHT CET 2021 (24 Sep Shift 1)

Options:

A. $2x + y - \frac{\pi}{2} - 1 = 0$

B. $2x-y-\frac{\pi}{2}+1=0$

C. $x+y-\frac{\pi}{2}-1=0$

D. $x-y-\frac{\pi}{2}+1=0$

Answer: A

Solution:

$$\begin{aligned} & y=\sqrt{2}\sin\left(2x+\frac{\pi}{4}\right) \\ & \frac{dy}{dx}=\sqrt{2}\cos\left(2x+\frac{\pi}{4}\right)(2) \\ & \therefore\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}}=(2\sqrt{2})\cos\left(\frac{\pi}{2}+\frac{\pi}{4}\right)=2\sqrt{2}\left(-\sin\frac{\pi}{4}\right)=-2 \end{aligned}$$

When $x=\frac{\pi}{4}, y=\sqrt{2}\sin\left(\frac{\pi}{2}+\frac{\pi}{4}\right)=1$Hence equation of required tangent is

$$(y-1)=-2\left(x-\frac{\pi}{4}\right)\Rightarrow 2x+y-\frac{\pi}{2}-1=0$$

Question219

For all real x, the minimum value of the function $f(x)=\frac{1-x+x^2}{1+x+x^2}$ is MHT CET 2021 (24 Sep Shift 1)

Options:

A. $\frac{1}{3}$

B. 0

C. 3

D. 1

Answer: A

Solution:

We have $f(x)=\frac{1-x+x^2}{1+x+x^2}$

$$f'(x)=\frac{\left(1+x+x^2\right)(2x-1)-\left(1-x+x^2\right)(2x+1)}{\left(1+x+x^2\right)^2}$$

$$=\frac{\left(2x+2x^2+2x^3-x-1-x^2\right)-\left(2x-2x^2+2x^3+1-x+x^2\right)}{\left(1+x+x^2\right)^2}$$

$$=\frac{\left(x+x^2+2x^3-1\right)-\left(x-x^2+2x^3+1\right)}{\left(1+x+x^2\right)^2}$$

$= \frac{2x^2-2}{(1+x+x^2)^2}$ and when $f'(x) = 0$, we get

$$2\left(x^2-1\right) = 0 \Rightarrow x = \pm 1$$

When $x = 1, f(x) = \frac{1}{3}$ and when $x = -1, f(x) = 3$Hence minimum value of $f(x)$ is $\frac{1}{3}$.

Question220

The radius of a circular plate is increasing at the rate of 0.01 cm/sec, when the radius is 12 cm. Then the rate at which the area increases is MHT CET 2021 (24 Sep Shift 1)

Options:

A. 60π sq. cm /sec

B. 0.24π sq. cm /sec

C. 1.2π sq. cm /sec

D. 24π sq. cm /sec

Answer: B

Solution:

We have $\frac{dr}{dt} = 0.01$

$$\begin{aligned} & A = \pi r^2 \\ \therefore \quad & \frac{dA}{dt} = \pi(2r)\frac{dr}{dt} = (2\pi)(12)(0.01) = 0.24\pi sq \cdot cm/sec \end{aligned}$$

Question221

10 is divided into two parts such that the sum of double of the first and square of the other is minimum, then the numbers are respectively MHT CET 2021 (23 Sep Shift 2)

Options:

A. 9,1

B. 8,2

C. 6,4

D. 7,3

Answer: A

Solution:

Let the two parts of 10 be x and $(10-x)$.

$$f(x) = 2(10-x) + x^2 = x^2 - 2x + 20$$

$f'(x) = 2x - 2$ and when $f'(x) = 0$, we get $x = 1$

$$f''(x) = 2 > 0$$

$\therefore$ f(x) is minimum at $x = 1$.Thus the parts are 1,9 .

Question222

Function $f(x) = e^{-1/x}$ is strictly increasing for all x where MHT CET 2021 (23 Sep Shift 2)

Options:

A. x is only positive real number

B. x is only negative real number

C. x is a real number

D. x is a non - zero real number

Answer: D

Solution:

$$f(x) = e^{-\frac{1}{x}}$$
$$f'(x) = e^{-\frac{1}{x}}(-1)\left(\frac{-1}{x^2}\right) = \frac{1}{x^2 e^{\frac{1}{x}}}$$

When $f'(x) > 0, x^2 e^{\frac{1}{x}} > 0$ and $x \neq 0$

Now $x^2 > 0$ and $e^{\frac{1}{x}} > 0$ for $\forall x \in R$

Hence $f(x)$ is an increasing function for $\forall x$, except $x = 0$

Question223

If $x = -2$ and $x = 4$ are the extreme points of $y = x^3 - \alpha x^2 - \beta x + 5$, then MHT CET 2021 (23 Sep Shift 2)

Options:

A. $\alpha = 3, \beta = 24$

B. $\alpha = -24 m \beta = -3$

C. $\alpha = -3, \beta = -24$

D. $\alpha = 24 \beta, \beta = 3$

Answer: A

Solution:

$$y=x^3-\alpha x^2-\beta x+5$$
$$\therefore \frac{dy}{dx}=3x^2-2\alpha x-\beta$$

We have $x=-2$ and $x=4$ as extreme points.

$$\therefore \left(\frac{dy}{dx}\right)_{x=-2}=3(-2)^2-2\alpha(-2)-\beta=0$$
$$\therefore 12+4\alpha-\beta=0 \Rightarrow 4\alpha-\beta=-12$$
$$\left(\frac{dy}{dx}\right)_{x=4}=3(4)^2-2\alpha(4)-\beta=0$$
$$\therefore 48-8\alpha-\beta=0 \Rightarrow 8\alpha+\beta=48$$

Solving eq. (1) and (2), we get $\alpha=3, \beta=24$

Question224

The maximum area of the rectangle that can be inscribed in a circle of radius r is MHT CET 2021 (23 Sep Shift 1)

Options:

A. $2r^2$ sq. units

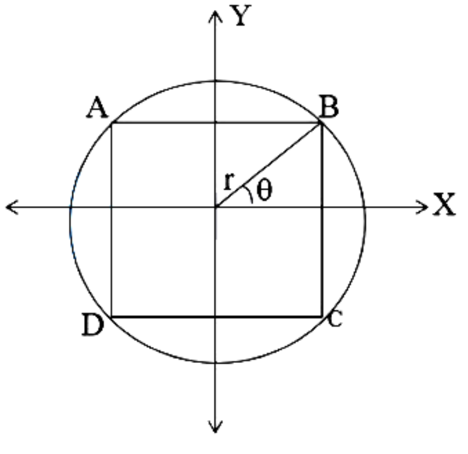
B. $\frac{\pi r^2}{4}$ sq. units

C. πr^2 units

D. r^3 sq units

Answer: A

Solution:



Refer figure

$$B = (r\cos\theta, r\sin\theta)$$
$$\therefore Ab2r\cos\theta \text{ and } BC = 2r\sin\theta$$
$$\text{Area }(ABCD) = AB \times BC$$
$$\therefore f(\theta) = (2r\cos\theta)(2r\sin\theta) = 2r^2\sin 2\theta$$
$$f'(\theta) = 4r^2\cos 2\theta \text{ and when } f'(\theta) = 0, \text{ we write}$$
$$\cos 2\theta = 0 \Rightarrow \cos 2\theta = \cos\frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}$$
$$f''(\theta) = -8r^2\sin 2\theta \text{ and } f''(\theta)_{[\theta=\frac{\pi}{4}]} = -8r^2 < 0\backslash$$

Thus, area of required rectangle is maximum when $\theta = \frac{\pi}{4}$

$$\therefore AB = 2r\cos\frac{\pi}{4} = 2r\left(\frac{1}{\sqrt{2}}\right) = \sqrt{2}r \text{ and}$$
$$BC = 2r\sin\frac{\pi}{4} = 2r\left(\frac{1}{\sqrt{2}}\right) = \sqrt{2}r$$

$\therefore$ Maximum Area $(ABCD) == (\sqrt{2}r)(\sqrt{2}r) = 2r^2$

Question225

$F(x) = \log|\sin x|$, **where** $x \in (0, \pi)$ **is strictly increasing on MHT CET 2021 (23 Sep Shift 1)**

Options:

A. $\left(\frac{\pi}{2}, \pi\right)$ only

B. $(0, \pi)$ only

C. $\left(0, \frac{\pi}{2}\right)$ only

D. $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ only

Answer: C

Solution:

$f(x) = \log|\sin x|$, where $x \in (0, \pi)$

$\therefore f'(x) = \frac{1}{\sin x} \times \cos x = \cot x$

When $f'(x) > 0$, we say $\frac{\cos x}{\sin x} > 0$

Here $\sin x > 0 \ldots [x \in (0, \pi)]$

for the function to be strictly increasing, $\cos x > 0$

$\rightarrow x \in \left(0, \frac{\pi}{2}\right)$ only.

Question226

A spherical raindrop evaporates at a rate proportional to its surface area. If its radius originally is $3 \mathrm{~mm}$. and 1 hour later has been reduced to $2 \mathrm{~mm}$, then the expression of radius r of the raindrop at any time t is (where $0 \leq t < 3$) MHT CET 2021 (22 Sep Shift 2)

Options:

A. $\mathrm{r}=\mathrm{t}+5$

B. $r=t-5$

C. $\mathrm{r}=3-\mathrm{t}$

D. $r=t+3$

Answer: C

Solution:

We have $\frac{\mathrm{dv}}{\mathrm{dt}} \propto-\left(4 \pi \mathrm{r}^2\right)$ We know that $\mathrm{v}=\frac{4}{3} \pi \mathrm{r}^2 \Rightarrow \frac{\mathrm{dv}}{\mathrm{dt}}=4 \pi \mathrm{r}^2 \frac{\mathrm{dr}}{\mathrm{dt}}$

$$\therefore 4 \pi \mathrm{r}^2 \frac{\mathrm{dr}}{\mathrm{dt}} \propto-\left(4 \pi \mathrm{r}^2\right)$$
$$\therefore 4 \pi \mathrm{r}^2 \frac{\mathrm{dr}}{\mathrm{dt}}=\left(-4 \mathrm{k} \pi \mathrm{r}^2\right) \Rightarrow \frac{\mathrm{dr}}{\mathrm{dt}}=-\mathrm{k}$$

$\therefore \mathrm{r}=-\mathrm{kt}+\mathrm{c}$ ... (1)

We have $\mathrm{r}=3$, when $\mathrm{t}=0$

$\therefore 3=\mathrm{c} \Rightarrow \mathrm{r}=-\mathrm{kt}+3$ ... (2)

We have $\mathrm{r}=2$, when $\mathrm{t}=1$

$\therefore 2=-\mathrm{k}+3 \Rightarrow \mathrm{k}=1$ ... (3)

$\therefore \mathrm{r}=-\mathrm{t}+3 \Rightarrow \mathrm{r}=3-\mathrm{t}$ ...[From (1), (2), (3)]

Question227

If $f(x)=x^2+a x+b$ has minima at $x=3$ whose value is 5 , then the values of a and b are respectively. MHT CET 2021 (22 Sep Shift 2)

Options:

A. -6,-14

B. -6,14

C. 14,-6

D. 6,14

Answer: B

Solution:

$$f(x) = x^2 + ax + b$$

$\therefore f'(x) = 2x + a$ and when $f'(x) = 0$, we get $x = \frac{-a}{2}$

Now $f''(x) = 2$ and $2 > 0$

$\therefore f(x)$ has minima at $x = \frac{-a}{2} = 3$ $\quad \ldots$ [as per given data]

$$\therefore a = -6$$

Since Minimum value of $f(x)$ is 5 at $x = 3$, we write

$$5 = (3)^2 + (-6)(3) + b \quad \Rightarrow b = 14$$

Question228

The function $f(x) = \frac{\lambda \sin x + 6 \cos x}{2 \sin x + 3 \cos x}$ is increasing, if MHT CET 2021 (22 Sep Shift 2)

Options:

A. $\lambda > 2$

B. $\lambda < 4$

C. $\lambda \geq 4$

D. $\lambda > 1$

Answer: A

Solution:

$$f(x) = \frac{\lambda \sin x + 6 \cos x}{2 \sin x + 3 \cos x}$$

$$f'(x) = \frac{\{[(2 \sin x + 3 \cos x)(\lambda \cos x - 6 \sin x] - [\lambda \sin x + 6 \cos x)(2 \cos x - 3 \sin x)]\}}{(2 \sin x + 3 \cos x)^2}$$

When $f'(x) \geq 0$, we get

$$\begin{aligned} &\left[\left(2\lambda \sin x \cos x + 3\lambda \cos^2 x - 12 \sin^2 x - 18 \sin x \cos x\right)\right. \\ &\left.- \left(2\lambda \sin x \cos x + 12 \cos^2 x - 3\lambda \sin^2 x - 18 \operatorname{sun} x \cos x\right)\right] \geq 0 \\ &\therefore 3\lambda \left(\sin^2 x + \cos^2 x\right) - 12 \left(\sin^2 x + \cos^2 x\right) \geq 0 \\ &\therefore 3\lambda - 12 \geq 0 \Rightarrow \lambda \geq 4 \end{aligned}$$

Question229

The slant height of a right circular cone is 3 cm. The height of the cone for maximum volume is MHT CET 2021 (22 Sep Shift 2)

Options:

A. 5 cm

B. $\sqrt{5}$ cm

C. 3 cm

D. $\sqrt{3}$ cm

Answer: D

Solution:

Volume of cone $= \frac{1}{3}\pi r^2$ hWe have slant height $\ell = 3$ cm and we know that $\ell^2 = r^2 + h^2$

$$\begin{aligned} &\therefore 9 = r^2 + h^2 \Rightarrow r^2 = 9 - h^2 \\ &\therefore v = \frac{1}{3}\pi\left(9 - h^2\right)h = (3\pi)h - \left(\frac{\pi}{3}\right)h^3 \\ &\frac{dv}{dh} = 3\pi - \left(\frac{\pi}{3}\right)\left(3\,h^2\right) = 3\pi - \pi h^2 \end{aligned}$$

When $\frac{dv}{dh} = 0$, we get $3\pi = \pi h^2 \Rightarrow h = \sqrt{3}$

$$\frac{d^2v}{dh^2} = 0 - 2\pi h \quad \Rightarrow \left(\frac{d^2v}{dh^2}\right)_{b-\sqrt{3}} = -2\sqrt{3}\pi < 0$$

$\therefore$ Volume of cone is maximum when $h = \sqrt{3}$

Question230

A rectangle of maximum area is inscribed in an ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, then its dimensions are MHT CET 2021 (22 Sep Shift 1)

Options:

A. $4\sqrt{2}, 6\sqrt{2}$

B. $\sqrt{2}, 5\sqrt{2}$

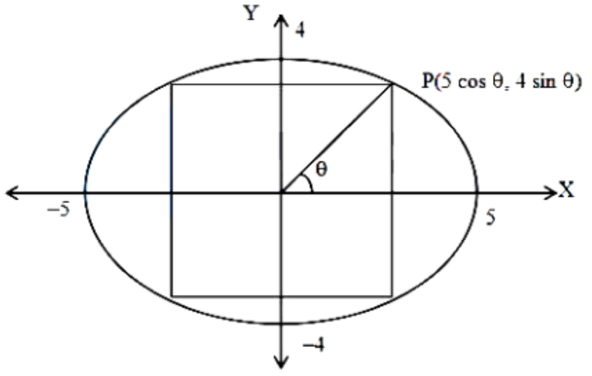
C. $4\sqrt{2}, 5\sqrt{2}$

D. $4\sqrt{2}, \sqrt{2}$

Answer: C

Solution:

Refer figure



Length of rectangle $= 10\cos\theta$ and

breadth of rectangle $= 8\sin\theta$

$\therefore$ Area of rectangle $= (10\cos\theta)(8\sin\theta) = 40(\sin\theta)$

Maximum area will occur when $\sin 2\theta = 1$

$\therefore \sin 2\theta = \sin\frac{\pi}{2} \quad \Rightarrow \theta = \frac{\pi}{4}$

$\therefore P = \left(\frac{5}{\sqrt{2}}, \frac{4}{\sqrt{2}}\right) \Rightarrow$ Dimensions of rectangle are $5\sqrt{2}, 4\sqrt{2}$

Question231

A spherical snow ball is forming so that its volume is increasing at the rate of $8\ \mathrm{cm}^3/\mathrm{sec}$. Find the rate of increase of radius when radius is $2\ \mathrm{cm}$ MHT CET 2021 (22 Sep Shift 1)

Options:

A. $\pi \mathrm{cm}/\mathrm{sec}$

B. $\frac{1}{8\pi} \mathrm{cm}/\mathrm{sec}$

C. $2\pi \mathrm{cm}/\mathrm{sec}$

D. $\frac{1}{2\pi} \mathrm{cm}/\mathrm{sec}$

Answer: D

Solution:

$\mathrm{V} = \frac{4}{3}\pi \mathrm{r}^3$

$\therefore \frac{\mathrm{dV}}{\mathrm{dt}} = \frac{4}{3}\pi\left(3\mathrm{r}^2\right)\frac{\mathrm{dr}}{\mathrm{dt}} = 4\pi \mathrm{r}^2\frac{\mathrm{dr}}{\mathrm{dt}}$

$\therefore 8 = 4\pi(2)^2\frac{\mathrm{dr}}{\mathrm{dt}} \Rightarrow \frac{\mathrm{dr}}{\mathrm{dt}} = \frac{1}{2\pi}$

Question232

The point on the curve $y^2 = 2(x-3)$ at which the normal is parallel to the line $y - 2x + 1 = 0$ is MHT CET 2021 (22 Sep Shift 1)

Options:

A. $\left(\frac{-1}{2}, -2\right)$

B. $\left(\frac{3}{2}, 2\right)$

C. $(5, 2)$

D. $(5, -2)$

Answer: D

Solution:

$$y^2 = 2(x-3)$$
$$\therefore 2y\frac{dy}{dx} = 2 \quad \Rightarrow \frac{dy}{dx} = \frac{1}{y}$$
$$\therefore \text{ Slope of normal } = -y \text{ and as per condition given}$$
$$-y = 2 \quad \Rightarrow y = -2$$
$$\therefore \quad (-2)^2 = 2(x-3) \Rightarrow x = 5 \Rightarrow \text{ point is } (5, -2)$$

Question233

For all real x, the minimum value of the function $f(x) = \frac{1-x+x^2}{1+x+x^2}$ is MHT CET 2021 (21 Sep Shift 2)

Options:

A. $\frac{1}{3}$

B. 0

C. 3

D. 1

Answer: A

Solution:

We have $f(x) = \frac{1-x+x^2}{1+x+x^2}$

$$f'(x) = \frac{\left(1+x+x^2\right)(2x-1) - \left(1-x+x^2\right)(2x+1)}{\left(1+x+x^2\right)^2}$$

$$= \frac{\left(2x+2x^2+2x^3-x-1-x^2\right) - \left(2x-2x^2+2x^3+1-x+x^2\right)}{\left(1+x+x^2\right)^2}$$

$$= \frac{\left(x+x^2+2x^3-1\right) - \left(x-x^2+2x^3+1\right)}{\left(1+x+x^2\right)^2}$$

$= \frac{2x^2-2}{\left(1+x+x^2\right)^2}$ and when $f'(x) = 0$, we get

$$2\left(\mathrm{x}^2-1\right) = 0 \Rightarrow \mathrm{x} = \pm 1$$

When $\mathrm{x}=1, \mathrm{f}(\mathrm{x})=\frac{1}{3}$ and when $\mathrm{x}=-1, \mathrm{f}(\mathrm{x})=3$Hence minimum value of $f(x)$ is $\frac{1}{3}$.

Question234

The function $\mathrm{f}(\mathrm{x})=\log(1+\mathrm{x})-\frac{2\mathrm{x}}{2+\mathrm{x}}$ is increasing on MHT CET 2021 (21 Sep Shift 2)

Options:

A. $(-\infty, \infty)$

B. $(-5, \infty)$

C. $(-\infty, 0)$

D. $(-1, \infty)$

Answer: D

Solution:

$$\begin{aligned} & \mathrm{f}(\mathrm{x})=\log(1+\mathrm{x})-\frac{2\mathrm{x}}{2+\mathrm{x}} \Rightarrow \mathrm{x} \neq -2 \\ & \therefore \mathrm{f}'(\mathrm{x})=\frac{1}{(1+\mathrm{x})}-\left[\frac{(2+\mathrm{x})(2)-(2\mathrm{x})(1)}{(2+\mathrm{x})^2}\right] \\ & \quad =\frac{1}{1+\mathrm{x}}-\left[\frac{4}{(2+\mathrm{x})^2}\right]=\frac{(\mathrm{x}+2)^2-4(\mathrm{x}+1)}{(\mathrm{x}+2)^2(1+\mathrm{x})} \end{aligned}$$

When $f'(x)>0$, we write

$$\frac{x^2}{(x+2)^2(1+x)}>0$$

Since $x^2>0$ and $(x+2)^2>0$, we write $(1+x)>0 \Rightarrow x>-1$

Question235

The abscissa of the points, where the tangent to the curve $y=x^3-3x^2-9x+5$ is parallel to X-axis are MHT CET 2021 (21 Sep Shift 2)

Options:

A. x=1 and -1

B. x=1 and -3

C. x=-1 and 3

D. x=0 and 1

Answer: C

Solution:

We have $y=x^3-3x^2-9x+5$

$\therefore \frac{dy}{dx}=3x^2-6x-9$ and since tangent is parallel to X axis, we write

$$3x^2-6x-9=0 \Rightarrow x^2-2x-3=0 \text{ i.e. } (x-3)(x+1)=0$$
$$\Rightarrow x=-1,3$$

Question236

A stone is dropped in a quiet lake and it is observed that waves move in circles, If the radius of a circular wave increases at the rate $2\ \mathrm{cm/sec}$, then the rate of increase in its area at the instant when its radius is $10\ \mathrm{cm}$, is $\mathrm{cm}^2/\mathrm{sec}$. MHT CET 2021 (21 Sep Shift 1)

Options:

A. 40π

B. 80π

C. 10π

D. 20π

Answer: A

Solution:

Given $\frac{dr}{dt}=2\ \mathrm{cm/sec} r=10\ \mathrm{cm}$

We have $A=\pi r^2$

Diff. w.r.t. t

$$\begin{aligned}\frac{dA}{dt} &= 2\pi r\frac{dr}{dt}\\ &= 2\pi\times 10\times 2\\ &= 40\pi\end{aligned}$$

Question237

The curves $\frac{x^2}{a^2}+\frac{y^2}{4}=4$ and $y^3=16x$ intersect each other orthogonally, then $\mathrm{a}^2=$ MHT CET 2021 (21 Sep Shift 1)

Options:

A. 2

B. $\frac{3}{4}$

C. $\frac{1}{2}$

D. $\frac{4}{3}$

Answer: D

Solution:

$$\frac{x^2}{a^2}+\frac{y^2}{4}=1$$
$$\therefore \frac{1}{a^2}2x+\frac{1}{4}2y\frac{dy}{dx}=0 \quad \Rightarrow \frac{dy}{dx}=\left(\frac{-x}{a^2}\right)\left(\frac{4}{y}\right)$$

Also $y^3=16x$

$$\therefore 3y^2\frac{dy}{dx}=16 \Rightarrow \frac{dy}{dx}=\frac{16}{3y^2}$$

Since curves intersect orthogonally, from (1) and (2), we write

$$\left(\frac{-x}{a^2}\right)\left(\frac{4}{y}\right)\left(\frac{16}{3y^2}\right)=-1$$

$\therefore \frac{64x}{3a^2y^3}=1$ and we have $y^3=16x$

$$\therefore \frac{64x}{3a^2(16x)}=1 \Rightarrow a^2=\frac{4}{3}$$

Question238

The function $f(x)=\cot^{-1}x+x$ is increasing in the interval. MHT CET 2021 (21 Sep Shift 1)

Options:

A. $(-\infty,\infty)$

B. $(0,3)$

C. $(1,\infty)$

D. $(-1,\infty)$

Answer: A

Solution:

$$f(x)=\cot^{-1}x+x$$
$$\therefore f'(x)=\frac{-1}{1+x^2}+1=\frac{-1+1+x^2}{1+x^2}=\frac{x^2}{1+x^2}$$
$$\text{Here } x^2\geq 0 \Rightarrow \frac{x^2}{1+x^2}\geq 0$$

Hence $f(x)$ is always increasing.

Question239

If $f(x)=2x^3-15x^2-144x-7$, then $f(x)$ is strictly decreasing in MHT CET 2021 (20 Sep Shift 2)

Options:

A. $(-8,3)$

B. $(-3,8)$

C. $(3,8)$

D. $(-8,-3)$

Answer: B

Solution:

$$\begin{aligned}&f(x)=2x^3-15x^2-144x-7\\&f'(x)=6x^2-30x-144\\&\text{When } f'(x)<0, \text{ we get } x^2-5x-24<0\\&\therefore (x-8)(x+3)<0 \Rightarrow -3<x<8\end{aligned}$$

Question240

Water is being poured at the rate of $36\ m^3/min$. into a cylindrical vessel, whose circular base is of radius $3\ m$. then the water level in the cylinder is rising at the rate of MHT CET 2021 (20 Sep Shift 2)

Options:

A. $\frac{4}{\pi} m/min$

B. $4\pi m/min$

C. $\frac{1}{4\pi} m/min$

D. $\frac{2}{\pi} m/min$

Answer: A

Solution:

Volume of cylinder $=\pi r^2 h$

$$\begin{aligned}&\therefore \frac{dv}{dt}=\pi r^2\frac{dh}{dt}\\&\therefore 36=\pi(3)^2\frac{dh}{dt}\Rightarrow\frac{dh}{dt}=\frac{4}{\pi}m/min\end{aligned}$$

Question241

The surface area of a spherical balloon is increasing at the rate $2\ cm^2/sec$. Then rate of increase in the volume of the balloon is , when the radius of the balloon is $6\ cm$. MHT CET 2021 (20 Sep Shift 2)

Options:

A. 4 $\mathrm{cm}^3/\mathrm{sec}$

B. 16 $\mathrm{cm}^3/\mathrm{sec}$

C. 36 $\mathrm{cm}^3/\mathrm{sec}$

D. 6 $\mathrm{cm}^3/\mathrm{sec}$

Answer: D

Solution:

We have $\frac{dA}{dt} = 2$ and $A = 4\pi r^2$

$$\therefore \frac{dA}{dt} = 8\pi r \frac{dt}{dt} \Rightarrow 2 = 8\pi(6)\frac{dr}{dt}$$
$$\therefore \frac{dr}{dt} = \frac{1}{24\pi}$$

Here $V = \frac{4}{3}\pi r^3$

$$\therefore \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$
$$= 4\pi(6)^2\left(\frac{1}{24\pi}\right) = 6\ \mathrm{cm}^3/\mathrm{sec}$$

Question242

A wire of length 20 units is divided into two parts such that the product of one part and cube of the other part is maximum, then product of these parts is MHT CET 2021 (20 Sep Shift 1)

Options:

A. 5

B. 75

C. 15

D. 70

Answer: B

Solution:

Let x be the one part and y be the other part.

We have $x + y = 20 \Rightarrow y = 20 - x$

As per condition given, we write

$$\begin{aligned} f(x) &= (20 - x)x^3 \\ &= 20x^3 - x^4 \\ \therefore f'(x) &= 60x^2 - 4x^3 \end{aligned}$$

When $f'(x)$, we get

$$\begin{aligned} &4x^2(15 - x) = 0 \Rightarrow x = 0, 15 \\ &f'(x) = 120x - 12x^2 \\ &[f'(x)]_{x=15} = (120)(15) - (12)(15)^2 = -900 < 0 \end{aligned}$$

$\therefore f[x]$ is maximum when $x = 15$.

$$\therefore y = 5 \Rightarrow xy = (15)(5) = 75$$

Question243

A particle is moving on a straight line. The distance S travelled in time t is given by $S = at^2 + bt + 6$. If the particle comes to rest after 4 seconds at a distance of 16 m. from the starting point, then the acceleration of the particle is. MHT CET 2021 (20 Sep Shift 1)

Options:

A. $\frac{-3}{4}$ m/sec^2

B. $\frac{-1}{2}$ m/sec^2

C. -1 m/sec^2

D. $\frac{-5}{4}$ m/sec^2

Answer: D

Solution:

$$\begin{aligned} &S = att^2 + bt + 6 \\ &\therefore V = \frac{dS}{dt} = 2at + b \text{ and } A = \frac{d^2 S}{dt^2} = 2a \end{aligned}$$

When particle comes to rest,

$$\begin{aligned} &S = 16, t = 4, \ V = 0 \\ &\therefore 16 = a(4)^2 + b(4) + 6 \\ &\Rightarrow 16a + 4b = 10 \ldots (1) \end{aligned}$$

Also $0 = 2a(1)(2) + 4$

$$\Rightarrow b = -8a$$

From (1) and (2), we get

$$16a + 4(8a) = 10 \Rightarrow -16 = 10 \Rightarrow a = \frac{-5}{9}$$

We have acceleration $A = 2a = 2\left(\frac{-5}{8}\right) = \frac{-5}{4}\ m/sec^2$

Question244

The equation of the tangent to curve $y = 4xe^x$ at $\left(-1, \frac{-4}{e}\right)$ is MHT CET 2021 (20 Sep Shift 1)

Options:

A. $6x - \frac{e}{4}y = -5$

B. $x - \frac{e}{4}y = 0$

C. $x = -1$

D. $y = \frac{-4}{e}$

Answer: D

Solution:

$$y = 4x^x$$
$$\therefore \left(\frac{dy}{dx}\right)_{\left(-1, \frac{-4}{e}\right)} = 4(-1)e^{-1} + 4e^{-1} = \frac{-4}{e} + \frac{4}{e} = 0$$

Thus tangent is parallel to X axis.Hence required equation of tangent is $y = \frac{-4}{e}$

Question245

If P is a point on the segment AB of length 12 cm, then the position of P for $AP^2 + BP^2$ to be minimum is such that MHT CET 2020 (20 Oct Shift 2)

Options:

A. P divides AB in the ratio 2 : 3 internally

B. P divides AB in the ratio 4 : 3 internally

C. P is the midpoint of segment AB

D. P divides BA in the ratio 2 : 1 internally

Answer: C

Solution:

$$\begin{aligned} \text{Let } d(AP) &= x \Rightarrow d(BP) = 12 - x \\ f(x) &= AP^2 + BP^2 \\ &= x^2 + (12 - x)^2 \\ &= 2x^2 - 24x + 144 \\ \therefore \quad f'(x) &= 4x - 24 \text{ and when } f'(x) = 0, \text{ we get } x = 6. \\ f''(x) &= 4 > 0 \end{aligned}$$

Hence $f(x)$ is minimum at $x = 6$

Question246

The approximate value of $(66)^{\frac{1}{3}}$ is MHT CET 2020 (20 Oct Shift 2)

Options:

A. 4.0416

B. 4.0447

C. 4.0433

D. 4.0481

Answer: A

Solution:

Let $a = 64,\ h = 2$ and let $f(x) = x^{\frac{1}{3}} \Rightarrow f'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3x^{\frac{2}{3}}}$

$\therefore f(a) = 64^{\frac{1}{3}} = 4 \quad$ and $\quad f'(a) = \frac{1}{3(64)^{\frac{2}{3}}} = \frac{1}{3(16)} = \frac{1}{48}$

We have $f(a + h) \neq f(a) + h \cdot f'(a)$

$\therefore (66)^{\frac{1}{3}} \div 4 + (2)\left(\frac{1}{48}\right) \div 4 + \frac{1}{24} \div 4.0416$

Question247

The function $f(x) = 3x^4 + 16x^3 - 30x^2 + 10$ is increasing for MHT CET 2020 (20 Oct Shift 2)

Options:

A. every real value of x

B. $x = 0, x = 1$ only

C. $x \in (-5, 0) \cup (1, \infty)$

D. $x \in [0, 1]$

Answer: C

Solution:

$f(x)=3x^4+16x^3-30x^2+10$

$\therefore f'(x)=12x^3+48x^2-60x$

When $f'(x)>0$, we write

$x\left(12x^2+48x-60\right)>0$

$12x\left(x^2+4x-5\right)>0$

$\therefore f'(x)>0$, when $x\in(-5,0)\cup(1,\infty)$

Question248

The approximate value of $f(x)=3x^2+5x+3$ at $x=3\cdot 02$ is MHT CET 2020 (20 Oct Shift 1)

Options:

A. 45.46

B. $v=45\cdot 46$

C. $44\cdot 76$

D. $44\cdot 46$

Answer: B

Solution:

Given $f(x)=3x^2+5x+3\Rightarrow f'(x)=6x+5$

Let $\mathrm{a}=3,\ \mathrm{h}=0.02$

$\therefore \mathrm{f(a)} \quad =\mathrm{f}(3)=27+15+3=45$

$f'(a)=f'(3) \quad =18+5=23$

We know that

$$\begin{aligned}\mathrm{f(a+h)} &\div \mathrm{f(a)+hf'(a)}\\ &=45+(0.02)(23)=45+0.46=45.46\end{aligned}$$

Question249

The minimum value of $f(x)=a^2\cos^2 x+b^2\sin^2 x$ if $a^2>b^2$, is MHT CET 2020 (20 Oct Shift 1)

Options:

A. a^2-b^2

B. b^2

C. a^2+b^2

D. a^2

Answer: D

Solution:

Given $f(x)$

$= a^2 \cos^2 x + b^2 \sin^2 x$

$= a^2 \left(\frac{1+\cos 2x}{2}\right) + b^2 \left(\frac{1-\cos 2x}{2}\right)$

$= \left(\frac{a^2+b^2}{2}\right) + \left(\frac{a^2-b^2}{2}\right) \cos 2x$

$f(x)$ will be maximum when $\cos 2x = 1$

f(x) will be minimum when $\cos 2\mathrm{x} = -1$

Hence minimum value of $f(x)$ is

$$\begin{aligned} f(x) &= \frac{a^2+b^2}{2} + \left(\frac{a^2-b^2}{2}\right)(-1) \\ &= \frac{a^2+b^2}{2} - \frac{a^2-b^2}{2} = b^2 \end{aligned}$$

Question250

If the line $y = 4x - 5$ touches the curve $y^2 = ax^3 + b$ at the point $(2, 3)$, then MHT CET 2020 (20 Oct Shift 1)

Options:

A. $a = -2, b = -7$

B. $a = -2, b = 7$

C. $a = 2 \quad, b = -7$

D. $a = 2, b = 7$

Answer: C

Solution:

Line $y=4x-5$ has slope 4

We have $y^2=ax^3+b \Rightarrow 2y\frac{dy}{dx}=3ax^2$

$$\therefore \frac{dy}{dx}=\frac{3ax^2}{2y}$$
$$\Rightarrow \left(\frac{dy}{dx}\right)_{(2,3)}=\frac{3a(2)^2}{2(3)}=2a$$

As per given condition, $2a=4 \Rightarrow a=2$

Now $y^2=ax^3+b$ at point $(2,3)$ becomes

$$9=8(2)+b \Rightarrow b=-7$$

This problem can be alternatively solved as follows:

Put $y=4x-5$ in $y^2=ax^3+b$

$\therefore (4x-5)^2=ax^3+b$

When $x=2$, we get

$9=8a+b\ldots(1)$

Now we will go by options

Put $a=2,\ b=-7$ in equation (1) we get

$8(a)-7=16-7=9$

Question251

The area of the square increases at the rate of $0.5\ \mathrm{cm}^2/\mathrm{sec}$. The rate at which itsperimeter is increasing when the side of the square is $10\ \mathrm{cm}$ long, is MHT CET 2020 (19 Oct Shift 2)

Options:

A. $0\cdot 3\ \mathrm{cm}/\mathrm{sec}$

B. $0\cdot 1\ \mathrm{cm}/\mathrm{sec}$

C. $0\cdot 2\ \mathrm{cm}/\mathrm{sec}$

D. $0\cdot 4\ \mathrm{cm}/\mathrm{sec}$

Answer: B

Solution:

Let x be the side of the square. Then area $A=x^2$

$\therefore \frac{dA}{dt}=2x\frac{dx}{dt}\Rightarrow 0.5=2(10)\frac{dx}{dt}$

$\therefore \frac{dx}{dt}=0.025$

Now perimeter $=4x=p$

$\therefore \frac{dp}{dt}=4\frac{dx}{dt}=4(0.025)=0.1\ \mathrm{cm/sec.}$

Question252

The equation of the normal to the curve $2x^2+y^2=12$ at the point $(2,2)$ is MHT CET 2020 (19 Oct Shift 2)

Options:

A. $2x-y+6=0$

B. $2x+y-6=0$

C. $x+2y+2=0$

D. $x-2y+2=0$

Answer: D

Solution:

Given equation of the curve is $2x^2+y^2=12$

$\therefore \quad 4x+2y\frac{dy}{dx}=0\Rightarrow \frac{dy}{dx}=\frac{-2x}{y}$

Slope of tangent at $(2,2)$ is $\frac{-2(2)}{2}-2\Rightarrow$ Slope of normal at $(2,2)$ is $\frac{1}{2}$

Equation of normal at $(2,2)$ is

$y-2=\frac{1}{2}(x-2)\Rightarrow 2y-4=x-2\Rightarrow x-2y+2=0$

Question253

The maximum volume of a right circular cylinder if the sum of its radius and height is $6\ \mathrm{m}$ is MHT CET 2020 (19 Oct Shift 2)

Options:

A. $16\pi m^3$

B. $32\pi m^3$

C. $4\pi m^3$

D. $64\pi m^3$

Answer: B

Solution:

Let height of cylinder be h and radius be $\mathrm{r} \Rightarrow \mathrm{r}+\mathrm{h}=6 \Rightarrow \mathrm{h}=6-\mathrm{r}$

Volume of cylinder $\mathrm{V}=\pi \mathrm{r}^2 \mathrm{~h}$

$=\pi \cdot r^2 \cdot(6-r)=\pi\left(6 r^2-r^3\right)$

For the maximum volume, $\frac{\mathrm{dV}}{\mathrm{dr}}=0$

$\pi\left(12 r-3 r^2\right)=0 \Rightarrow 12 r=3 r^2 \Rightarrow r=4 \Rightarrow h=6-4=2$.

$\therefore$ Volume of cylinder $=\pi \times 4^2 \times 2=32 \pi \cdot \mathrm{m}^3$

Question254

The function $f(x)=(x+2) e^{-x}$ is MHT CET 2020 (19 Oct Shift 1)

Options:

A. decreasing in $(-\infty,-1)$ and increasing in $(-1, \infty)$

B. decreasing for all x

C. increasing in $(-\infty,-1)$ and decreasing in $(-1, \infty)$

D. increasing for all x

Answer: C

Solution:

Given

$$\begin{aligned} & f(x)=(x+2) e^{-x} \\ & \therefore f^{\prime}(x)=(x+2)\left(e^{-x}\right)(-1)+e^{-x}(1) \\ & \qquad=e^{-x}[1-(x+2)]=e^{-x}(-x-1) \\ & \qquad=-e^{-x}(x+1)\end{aligned}$$

Here e^{-x} is always positive.

$$\begin{aligned} & \text{Now } -(x+1)>0 \Rightarrow-x>1 \Rightarrow x<-1 \\ & \text{Also } -(x+1)<0 \Rightarrow-x<1 \Rightarrow x>-1\end{aligned}$$

Thus $\mathrm{f}(\mathrm{x})$ increases in $(-\infty,-1)$ and decreases in $(-1, \infty)$

Question255

If L.M.V.T. is applicable for the function $f(x)=x+\frac{1}{x}, x \in[1,3]$, then $c=$ MHT CET 2020 (19 Oct Shift 1)

Options:

A. $-\sqrt{3}$

B. $\sqrt{3}$

C. 2

D. $\sqrt{2}$

Answer: B

Solution:

Given $\mathrm{f(x)} = \mathrm{x} + \frac{1}{\mathrm{x}}$ and LMVT holds

$$f'(x) = 1 - \frac{1}{x^2} \Rightarrow f'(c) = 1 - \frac{1}{c^2}$$
$$f(1) = 1 + 1 = 2 \text{ and } f(3) = 3 + \frac{1}{3} = \frac{10}{3}$$

$$\therefore \quad f'(c) = 1 - \frac{1}{c^2} = \frac{\frac{10}{3} - 2}{(3-1)} \Rightarrow 1 - \frac{1}{c^2} = \frac{4}{3(2)} = \frac{2}{3}$$

$$\therefore \quad \frac{1}{c^2} = 1 - \frac{2}{3} = \frac{1}{3} \Rightarrow c^2 = 3 \Rightarrow c = \pm\sqrt{3}$$

Question256

The equation of normal to the curve $\mathrm{y} = \sin\left(\frac{\pi x}{4}\right)$ at the point $(2, 5)$ is MHT CET 2020 (16 Oct Shift 2)

Options:

A. x+y=5

B. y=5

C. x=2

D. x+y=2

Answer: C

Solution:

we have, $y = \sin \frac{\pi x}{4}$

$$\frac{dy}{dx} = \frac{\pi}{4} \cdot \cos \frac{\pi x}{4}$$
$$\left(\frac{dy}{dx}\right)_{(2,5)} = \frac{\pi}{4} \cos \frac{2\pi}{4} = 0$$

Since slope of tangent is zero, it is parallel to X axis. So normal is parallel to Y axis. Hence required equation of normal is $\mathrm{x} = 2$

Question257

For every value of x, the function $f(x) = \frac{1}{a^x}, a > 0$ is MHT CET 2020 (16 Oct Shift 2)

Options:

A. decreasing

B. increasing

C. Constant

D. Neither increasing nor decreasing

Answer: A

Solution:

$$f(x) = \frac{1}{a^x} = a^{-x}$$

$$\therefore \quad f'(x) = -a^{-x} \cdot \log_e a = -\frac{\log_e a}{a^x} =< 0$$

So $f(x)$ is decreasing for all x.

Question258

If $x + y = \frac{\pi}{2}$, then the maximum value of $\sin x$.siny is MHT CET 2020 (16 Oct Shift 2)

Options:

A. $\frac{1}{2}$

B. $\frac{-1}{2}$

C. $\frac{-1}{\sqrt{2}}$

D. $\frac{1}{\sqrt{2}}$

Answer: A

Solution:

$$x + y = \frac{\pi}{2} \quad \Rightarrow \quad y = \frac{\pi}{2} - x$$

$$\sin x \cdot \sin y = \sin x \cdot \sin\left(\frac{\pi}{2} - x\right) = \sin x \cos x = \frac{2 \sin x \cdot \cos x}{2} = \frac{\sin 2x}{2}$$

$$\text{We know,} \ -1 \leq \sin 2x \leq 1 \Rightarrow \frac{-1}{2} \leq \frac{\sin 2x}{2} \leq \frac{1}{2}$$

So maximum value is $\frac{1}{2}$

Question259

If $f(x) = \log(\sin x), x \in \left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$, then value of 'c'by applyng L.mv.T. is MHT CET 2020 (16 Oct Shift 1)

Options:

A. $\frac{\pi}{2}$

B. $\frac{2\pi}{3}$

C. $\frac{3\pi}{4}$

D. $\frac{\pi}{4}$

Answer: A

Solution:

Value of ' ' by applying L.M.V.T. is to be found out

$$f(x)=\log(\sin x) \text{ on } \left[\frac{\pi}{6},\frac{5\pi}{6}\right] \Rightarrow \text{lot } a=\frac{\pi}{6}, b=\frac{5\pi}{6}$$
$$f'(x)=\frac{\cos x}{\sin x}=\cot x \Rightarrow f'(c)=\cot c$$
$$f'(c)=\frac{f(b)-f(a)}{b-a}=\frac{f\left(\frac{5\pi}{6}\right)-f\left(\frac{\pi}{6}\right)}{5\frac{\pi}{6}-\frac{\pi}{6}}$$
$$\cot c=\frac{\log\left[\sin\left(\frac{5\pi}{6}\right)\right]-\log\left[\sin\left(\frac{\pi}{6}\right)\right]}{\frac{2\pi}{3}}=\frac{\log\frac{1}{2}-\log\left(\frac{1}{2}\right)}{2\frac{\pi}{3}}$$
$$\cot c=0 \Rightarrow c=\frac{\pi}{2}$$

Question260

The equation of tangent at $\mathrm{P}(-4,-4)$ on the curve $x^2=-4y$ is MHT CET 2020 (16 Oct Shift 1)

Options:

A. 2x+y+4=0

B. 2x-y+4=0

C. 2x+y-4=0

D. 3x-y+8=0

Answer: B

Solution:

We have $x^2=-4y \Rightarrow 2x=-4\frac{dy}{dx} \Rightarrow \frac{dy}{dx}=\frac{-x}{2}$

Hence slope of tangent at $P(-4,-4)$ is 2 . Hence required eq. of tangent is
$y+4=2(x+4) \quad \Rightarrow 2x-y+4=0$

Question261

The approximate value of the function $f(x)=x^3-3x+5$ at $x=1.99$ is MHT CET 2020 (16 Oct Shift 1)

Options:

A. 6.91

B. 6.94

C. 7.94

D. 7.91

Answer: A

Solution:

$f(x)=x^3-3x+5 \Rightarrow f'(x)=3x^2-3$

Let $a=2, h=-0.01$

$\therefore f(a)=f(2)=2^3-3(2)+5=7$

$f'(a)=f'(2)=3(2)^2-3=9$

We know that $f(a+h) \neq f(a)+hf'(a)$

$f(1.99) \div 7-(0.01)(9) \div 7-0.09 \neq 6.91$

Question262

The approximate value of the function $\mathrm{f}(x)=x^3+5x^2-7x+10$ at $x=1 \cdot 1$ is MHT CET 2020 (15 Oct Shift 2)

Options:

A. $7 \cdot 6$

B. $8 \cdot 6$

C. $6 \cdot 6$

D. $9 \cdot 6$

Answer: D

Solution:

Given $f(x)=x^3+5x^2-7x+10 \Rightarrow f'(x)=3x^2+10x-7$

Let $a=1, h=0.1$

$\therefore \mathrm{f}(1)=1+5-7+10=9$ and $\mathrm{f}'(1)=3+10-7=6$

We know that $\mathrm{f}(\mathrm{a}+\mathrm{h}) \div \mathrm{f}(\mathrm{a})+\mathrm{hf}'(\mathrm{a})$

$f(1.1)=9+(0.1)(6)=9+0.6=9.6$

Question263

If the tangent to the curve given by $x=t^2-1$ and $y=t^2-t$ is parallel to X - axis, then the value of t is MHT CET 2020 (15 Oct Shift 2)

Options:

A. $\frac{-1}{\sqrt{3}}$

B. 0

C. $\frac{1}{\sqrt{3}}$

D. $\frac{1}{2}$

Answer: D

Solution:

We have $x=t^2-1$ and $y=t^2-t$

$$\therefore \frac{\mathrm{dx}}{\mathrm{dt}}=2\mathrm{t} \text{ and } \frac{\mathrm{dy}}{\mathrm{dt}}=2\mathrm{t}-1$$

$$\therefore \frac{\mathrm{dy}}{\mathrm{dx}}=\frac{\left(\frac{\mathrm{dy}}{\mathrm{dt}}\right)}{\left(\frac{\mathrm{dx}}{\mathrm{dt}}\right)}=\frac{2\mathrm{t}-1}{2\mathrm{t}}$$

Since tangent is parallel to X axis, we write

$$\frac{2t-1}{2t}=0 \Rightarrow t=\frac{1}{2}$$

Question264

If rectangles are inscribed in a circle of radius r units. Then the dimensions of the rectangle which has maximum area are MHT CET 2020 (15 Oct Shift 2)

Options:

A. 2r units, r units,

B. 2r units, $\sqrt{2}$r units,

C. r units, $\sqrt{2}$r units,

D. $\sqrt{2}$r units, $\sqrt{2}$r units

Answer: D

Solution:

Let $ABCD$ be the rectangle inscribed in a circle of radius 'r'.

$\Rightarrow \mathrm{AC}=\mathrm{BD}=2\mathrm{r}=$ diameter

Let x and y be the length and breath of rectangle.

$\therefore x^2+y^2=(2r)^2 \Rightarrow y=\sqrt{4r^2-x^2}$

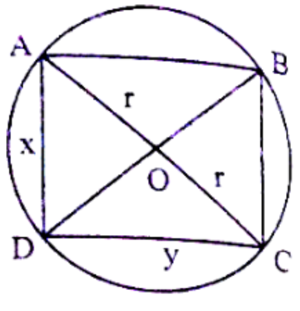
Now Area of rectangle $=\mathrm{A}=\mathrm{xy}$

$$\therefore A=x\sqrt{4r^2-x^2}$$
$$\therefore \frac{dA}{dx}=\sqrt{4r^2-x^2}+\frac{x}{2\sqrt{4r^2-x^2}}\times(-2x)=\sqrt{4r^2-x^2}-\frac{x^2}{\sqrt{4r^2-x^2}}$$
$$\frac{dA}{dx}=\frac{4r^2-2x^2}{\sqrt{4r^2-x^2}}$$

For maximum Area, $\frac{\mathrm{dA}}{\mathrm{dx}}=0 \Rightarrow 4\mathrm{r}^2-2\mathrm{x}^2=0 \Rightarrow \mathrm{x}=\sqrt{2}\mathrm{r}$

$$\frac{\mathrm{d}^2\mathrm{A}}{\mathrm{dx}^2}>0 \text{ for } \mathrm{x}=\sqrt{2}\mathrm{r}$$

Therefore area is maximum when $x=\sqrt{2}r \Rightarrow y=\sqrt{4r^2-2r^2}=\sqrt{2}r$



Question265

If line $x+y=0$ touches the curve $ax^2=2y^2-b$ at $(1,-1)$,then the values of a and b are respectively MHT CET 2020 (15 Oct Shift 1)

Options:

A. $0,2$

B. $-2,0$

C. $0,-2$

D. $2,0$

Answer: D

Solution:

$$ax^2 = 2y^2 - b$$
$$a \times 2x = 4y\frac{dy}{dx} - 0 \Rightarrow ax = 2y\frac{dy}{dx}$$
$$\therefore \frac{dy}{dx} = \frac{ax}{2y}$$

At $(1, -1)$ slope of tangent $= \frac{dy}{dx} = \frac{-a}{2}$ and slope of $x + y = 0$ is -1

As per condition given, $\frac{-a}{2} = -1 \Rightarrow a = 2$

Substituting $x = 1, y = -1, a = 2$ in the given equation of curve, we get

$$2 = 2 - b \Rightarrow b = 0$$

Question266

20 meters wire is available to fence a flower bed in the form of a circular sector. If the flower bed should have the greatest possible surface area, then the radius of the circle is MHT CET 2020 (15 Oct Shift 1)

Options:

A. 2 m

B. 4 m

C. 5 m

D. 10 m

Answer: C

Solution:

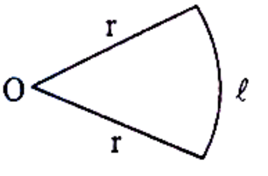
Let ℓ and r be as shown in figure. We have $20 = 2r + \ell$

$$\ell = 20 - 2r$$

Area of flower bed

$$A = \frac{1}{2} \times \ell r = \frac{1}{2}(20 - 2r) \cdot r$$
$$A = 10r - r^2$$

$$\frac{dA}{dr} = 10 - 2r \text{ and } \frac{d^2A}{dr^2} = -2 < 0$$
$$\text{When } \frac{dA}{dr} = 0 \Rightarrow 10 - 2r = 0 \Rightarrow r = 5$$



Hence area of flower bed will be maximum when $r = 5$.

Question267

The equation of normal to the curve $2x^2 + 3y^2 - 5 = 0$ at $P(1, 1)$ is MHT CET 2020 (15 Oct Shift 1)

Options:

A. $3x + 2y + 1 = 0$

B. $3x - 2y + 1 = 0$

C. $3x + 2y - 5 = 0$

D. $3x - 2y - 1 = 0$

Answer: D

Solution:

Given $2x^2 + 3y^2 - 5 = 0$

$$4x + 6y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-2x}{3y}$$

At$(1, 1)$, $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{-2}{3} \Rightarrow$ Slope of normal $= \frac{3}{2}$ Equation of normal is

$$y - 1 = \frac{3}{2}(x - 1) \Rightarrow 2y - 2 = 3x - 3$$
$$\therefore \quad 3x - 2y - 1 = 0$$

Question268

If the displacement of a particle at a point is given by $s = 3t^2 - 12t + 14$, then thedisplacement of the particle when its velocity becomes zero is MHT CET 2020 (14 Oct Shift 2)

Options:

A. 14 units

B. 4 units

C. 0 units

D. 2 units

Answer: D

Solution:

Given $\quad s \quad = 3t^2 - 12t + 14$

$$v = \frac{ds}{dt} = 6t - 12$$

When $v = 0$, we get $t = 2 \therefore \mathrm{s} = (3 \times 4) - (12 \times 2) + 14 = 12 - 24 + 14 = 2$ units

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Question269

The approximate value of $\log_{10} 99$ is (Given $\log_{10} e = 0.4343$) MHT CET 2020 (14 Oct Shift 2)

Options:

A. 1.9657

B. 1.9857

C. 1.1.9957

D. 1.9757

Answer: C

Solution:

Let $f(x) = \log_{10} x = \frac{\log_e x}{\log_e 10}$

$f'(x) = \frac{1}{x \log 10}$

Let $a = 100,\ h = -1$

$\therefore f(a) = \log_{10} 100 \quad = \log_{10} 10^2 = 2$

$f'(a) = \frac{1}{100 \times \log_e 10} = \frac{1}{100} \log_{10} e = \frac{1}{100} \times 0.4343$

We know that

$$\begin{aligned} f(a-h) &\doteqdot f(a) + hf'(a) \\ &\doteqdot 2 + (-1) \times \frac{1}{100}(0.4343) = 2 - 0.004343 = 1.995657 \\ &\doteqdot 1.9957 \end{aligned}$$

Question270

The function $f(x) = \log x - \frac{2x}{x+2}$ MHT CET 2020 (14 Oct Shift 2)

Options:

A. $x \in (-\infty, 1)$

B. $x \in (-1, \infty)$

C. $x \in (-\infty, 0)$

D. $x \in (0, \infty)$

Answer: D

Solution:

$$
\begin{aligned}
\text{Given } f(x) &= \log x - \frac{2x}{x+2} \\
\therefore f'(x) &= \frac{1}{x} - \left[\frac{(x+2)(2)-(2x)(1)}{(x+2)^2}\right] \\
&= \frac{1}{x} - \left[\frac{4}{(x+2)^2}\right] = \frac{x^2+4x}{x(x+2)^2} \\
&= \frac{x(x+4)}{x(x+2)^2}
\end{aligned}
$$

For $f'(x) > 0, x \neq 0$ and $x > -4$Hence $x \in (0, \infty)$ is permissible.

Question271

A particle moves according to the law $s = t^3 - 6t^2 + 9t + 25$. The displacement of MHT CET 2020 (14 Oct Shift 1)

Options:

A. 0 units

B. -27 units

C. 27 units

D. 9 units

Answer: C

Solution:

Given $s = t^3 - 6t^2 + 9t + 25$ ....(1)

$$
\begin{aligned}
v &= \tfrac{ds}{dt} = 3t^2 - 12t + 9 \\
\tfrac{dv}{dt} &= 6t - 12
\end{aligned}
$$

Given $6t - 12 = 0 \Rightarrow t = 2 \therefore$ from $(1), s = (2)^3 - 6 \times 4 + 18 + 25 = 27$

Question272

A metal wire 108 meters long is bent to form a rectangle. If the area of the rectangleis maximum, then its dimensions are MHT CET 2020 (14 Oct Shift 1)

Options:

A. 28 m, 28 m

B. 27 m, 27 m

C. 25 m, 25 m

D. 26 m, 26 m

Answer: B

Solution:

Let sides of rectangle be x and y

Thus $2x+2y=108 \Rightarrow x+y=54 \Rightarrow y=54-x$

Now Area $= A = xy$

$\therefore \quad = x(54-x) = 54x - x^2$

Differentiating w.r.t. $\mathbf{x}$, we get

$\frac{\mathrm{dA}}{\mathrm{dx}} = 54 \times 1 - 2\mathrm{x}$ and $\frac{\mathrm{d}^2\mathrm{A}}{\mathrm{dx}^2} = -2 < 0$

When $\frac{dA}{dx} = 0$, we get $54-2x=0 \Rightarrow x=27 \Rightarrow y=54-x=27$

Question273

The approximate value of $\cot^{-1}(1 \cdot 001)$ is MHT CET 2020 (13 Oct Shift 2)

Options:

A. $\frac{\pi}{4} - 0 \cdot 0005$

B. $\frac{\pi}{4} + 0 \cdot 005$

C. $\frac{\pi}{4} + 0 \cdot 0005$

D. $\frac{\pi}{4} - 0 \cdot 005$

Answer: A

Solution:

Let $f(x) = \cos^{-1} x \Rightarrow f'(x) = \frac{-1}{1+x^2}$

Let, $\mathrm{a}=1,\ \mathrm{h}=0.001$

Now $\mathrm{f(a)} = \cot^{-1} 1 = \frac{\pi}{4}$ and $\mathrm{f'(a)} = \frac{-1}{1+1} = -\frac{1}{2}$

We know that, $f(a+h) \doteqdot f(a) + hf'(a)$

$= \frac{\pi}{4} + (0.001)\left(-\frac{1}{2}\right) = \frac{\pi}{4} - 0.0005$

Question274

If the line $6x-y-4=0$ touches the curve $y^2 = ax^3 + b$ at the point $(1,2)$ then$a+b=$ MHT CET 2020 (13 Oct Shift 2)

Options:

A. 8

B. -4

C. 4

D. 12

Answer: C

Solution:

Slope of line $6x - y - 4 = 0$ is 6 and this line is tangent to the curve $y^2 = ax^3 + b$ at point $(1, 2)$

$\therefore 2y\frac{dy}{dx} = 3ax^2 \Rightarrow \left(\frac{dy}{dx}\right)_{(1,2)} = \frac{3ax^2}{2y} = \frac{3a}{4}$ and $\frac{3a}{4} = 6 \Rightarrow a = 8$

Now point $(1, 2)$ lies on given curve.

$\therefore (2)^2 = (8)(1)^3 + b \Rightarrow b = -4 \Rightarrow a + b = 8 - 4 = 4$

Question275

The maximum value of the function $\frac{\log x}{x}, x \neq 0$ is MHT CET 2020 (13 Oct Shift 2)

Options:

A. e^2

B. $\frac{1}{e}$

C. $\frac{1}{e^2}$

D. e

Answer: B

Solution:

Let $y = \frac{\log x}{x}$

$\therefore \frac{dy}{dx} = \frac{x \cdot \frac{1}{x} - \log x}{x^2} = \frac{1 - \log x}{x^2}$

Put $\frac{dy}{dx} = 0$, we get

$1 - \log x = 0 \Rightarrow \log x = 1 \Rightarrow \log x = \log e \Rightarrow x = e$

$$\text{Now } \frac{d^2y}{dx^2} = \frac{x^2\left(-\frac{1}{x}\right) - (1 - \log x)(2x)}{x^4} = \frac{-x - 2x + 2x\log x}{x^4}$$
$$= \frac{2\log x - 3}{x^3}$$

At $x = e$, $\frac{d^2y}{dx^2} < 0$

from (1), maximum value is $y = \frac{1}{e}$

Question276

A tangent to the curve $x = at^2, y = 2at$ is perpendicular to X axis, then the pointof contact is MHT CET 2020 (13 Oct Shift 1)

Options:

A. $(0,-a)$

B. $(0,0)$

C. $(0,2a)$

D. $(0,a)$

Answer: B

Solution:

Given equation of curve represents parabola $y^2 = 4ax$

Given is that tangent is $\perp$ er to X-axis. Therefore point of contact is vertex of parabola which is origin.

This problem can also be solved as

We have $x = at^2$ and $y = 2$ at

$\therefore \frac{dx}{dt} = 2at$ and $\frac{dy}{dt} = 2a$

$\therefore \frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t} \Rightarrow$ Slope of tangent $= \frac{1}{t}$. Since tangent is Perpendicular to X axis, it is parallel to Y axis i.e. it's slope is indefinite. It means $t = 0 \Rightarrow x = 0$ and $y = 0$

Question277

The radius of a circle is increasing at the rate $2\ \mathrm{cm/sec}$. The rate at which its area isincreasing when the radius of the circle is 5 decimeters is MHT CET 2020 (13 Oct Shift 1)

Options:

A. $100\pi \mathrm{cm}^2/\mathrm{sec}$

B. $200\pi \mathrm{cm}^2/\mathrm{sec}$

C. $2000\pi \mathrm{cm}^2/\mathrm{sec}$

D. $20\pi \mathrm{cm}^2/\mathrm{sec}$

Answer: B

Solution:

Here $\frac{dr}{dt} = 2$ and $r = 5$ decimeter $= 50$ cm Now Area of circle $= A = \pi r^2$.

$$\begin{aligned}\therefore \frac{dA}{dt} &= 2\pi r \frac{dr}{dt} \\ &= 2 \times \pi \times 50 \times 2 = 200\pi \mathrm{cm}^2/\mathrm{sec}\end{aligned}$$

Question278

The equation of a normal to the curve $x = 4\sec\theta$ and $y = 4\tan^2\theta$ at $\theta = \frac{\pi}{4}$ is MHT CET 2020 (13 Oct Shift 1)

Options:

A. $x+y\sqrt{2}=7\sqrt{2}$

B. $2\sqrt{2}x+y=8\sqrt{2}$

C. $\sqrt{2}x+y=7\sqrt{2}$

D. $x+2\sqrt{2}y=12\sqrt{2}$

Answer: D

Solution:

$$\begin{aligned} & x=4\sec\theta \quad \text{and} \quad y=4\tan^2\theta \\ \therefore \quad & \frac{dx}{d\theta}=4\sec\theta\tan\theta \quad \text{and} \quad \frac{dy}{d\theta}=8\tan\theta\cdot\sec^2\theta \end{aligned}$$

$\therefore \frac{dy}{dx}=\text{slope of tangent}=\frac{8\tan\theta\sec^2\theta}{4\sec\theta\tan\theta}=2\sec\theta$

At $\theta=\frac{\pi}{4}$, $\frac{dy}{dx}=2\sqrt{2}\Rightarrow\text{slope of normal}=-\frac{1}{\left(\frac{dy}{dx}\right)}=\frac{-1}{2\sqrt{2}}$

At $\theta=\frac{\pi}{4}$, $x=4\sqrt{2}$ and $y=4$

Hence eq. of normal is

$$\begin{aligned} & (y-4)=\frac{-1}{2\sqrt{2}}(x-4\sqrt{2}) \\ \therefore \quad & 2\sqrt{2}(y-4)=-x+4\sqrt{2}\Rightarrow x+2\sqrt{2}y=12\sqrt{2} \end{aligned}$$

Question279

If the L. M. V. T. holds for the function $f(x)=x+\frac{1}{x}, x\in[1,3]$, then c= MHT CET 2020 (12 Oct Shift 2)

Options:

A. $\sqrt{3}$

B. 3

C. 2

D. $-\sqrt{3}$

Answer: A

Solution:

Given $f(x)=x+\frac{1}{x}$ and LMVT holds

$f'(x)=1-\frac{1}{x^2}\Rightarrow f'(c)=1-\frac{1}{c^2}$

$f(1)=1+1=2$ and $f(3)=3+\frac{1}{3}=\frac{10}{3}$

$\therefore \quad f'(c)=1-\frac{1}{c^2}=\frac{\frac{10}{3}-2}{(3-1)}\Rightarrow 1-\frac{1}{c^2}=\frac{4}{3(2)}=\frac{2}{3}$

$\therefore \quad \frac{1}{c^2}=1-\frac{2}{3}=\frac{1}{3}\Rightarrow c^2=3\Rightarrow c=\pm\sqrt{3}$

Question280

The perimeter of a triangle is $10 \mathrm{~cm}$. If one of its side is $4 \mathrm{~cm}$, then remaining sides of the triangle, when area of triangle is maximum are MHT CET 2020 (12 Oct Shift 2)

Options:

A. 5 cm, 1 cm

B. $3 \cdot 6$ cm, $2 \cdot 4$ cm

C. 3 cm, 3 cm

D. 2 cm, 4 cm

Answer: C

Solution:

Let $\mathrm{a}, \mathrm{b}, \mathrm{c}$ be the sides of the triangle.

The perimeter of triangle $(2 \mathrm{~s})=\mathrm{a}+\mathrm{b}+\mathrm{c}$

Let $\mathrm{a}=4$ and $2 \mathrm{~s}=10$ i.e. $\mathrm{s}=5$

$\therefore 10=4+b+c \Rightarrow b=6-c$

Now, area of triangle $=\Delta=\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{5(5-4)(5-6+c)(5-c)}$

$\Delta=\sqrt{5(1)(\mathrm{c}-1)(5-\mathrm{c})} \Rightarrow \Delta^2=5(\mathrm{c}-1)(5-\mathrm{c})$

$=5\left(5 c-c^2-5+c\right)=5\left(-c^2+6 c-5\right)$

Let $f(c)=5\left(-c^2+6 c-5\right) \Rightarrow f^{\prime}(c)=5(-2 c+6) \Rightarrow f^{\prime \prime}(c)=5(-2)=-10$

For extreme value, $f^{\prime}(c)=0$ i.e. $5(-2 c+6)=0 \Rightarrow c=3$

Thus $f^{\prime \prime}(3)=-10<0 \Rightarrow f(c)$ has maximum value at $c=3$

$\therefore \mathrm{b}=6-3=3$ i.e. the lengths of the remaining sides are $3 \mathrm{~cm}$ and $3 \mathrm{~cm}$.

Question281

The displacement of a particle at time 't' is $s=t^3-4 t^2-5 t$, then the velocity ofthe particle at $t=2 \mathrm{sec}$. is MHT CET 2020 (12 Oct Shift 2)

Options:

A. $\left(\frac{1}{9}\right)$ units /sec.

B. -9 units /sec

C. 9 units /sec.

D. $\left(\frac{-1}{9}\right)$ units / sec.

Answer: B

Solution:

Given $s = t^3 - 4t^2 - 5t$

$\therefore \frac{ds}{dt} = 3t^2 - 8t - 5$

At $t = 2, \quad v = \frac{ds}{dt} = 3(4) - 8(2) - 5 = 12 - 16 - 5 = -9$

Question282

If $f(x) = |x-2|, x \in [0,4]$ then the Rolle's theorem cannot be applied to thefunction because MHT CET 2020 (12 Oct Shift 1)

Options:

A. The function is not differentiable at every point in the $(0,4)$.

B. $f(4) \neq f(0)$

C. Function is not well-defined in the domain.

D. The function is not continuous at every point in the $[0,4]$.

Answer: A

Solution:

Here $f(0) = |0-2| = -2$ and $f(4) = |4-2| = 2$

Thus $\mathrm{f}(4) \neq \mathrm{f}(0)$

Hence Rolle's Theorem cannot be applied.

Question283

A stone is dropped into a pond. Waves in the form of circles are generated and radius of outermostripple increases at the rate of $5\mathrm{cm}/\mathrm{sec}$. Then area increased after 2 seconds is ________ MHT CET 2019 (02 May Shift 1)

Options:

A. $100\pi\mathrm{cm}^2/\mathrm{sec}$

B. $40\mathrm{cm}^2/\mathrm{sec}$

C. $50\mathrm{cm}^2/\mathrm{sec}$

D. $25\mathrm{cm}^2/\mathrm{sec}$

Answer: A

Solution:

Given: $\frac{dr}{dt} = 5\mathrm{cm}/\mathrm{sec} \Rightarrow dr = 5dt$
then $r = 5t + c$, at $t = 0, \Rightarrow r = 0 \Rightarrow c = 0$
$r = 5t$
New $A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$= 2\pi . 10.5$ at $t = 2 \Rightarrow r = 10$
$= 100\pi \ cm^2/\text{sec}.$

Question284

The equation of normal to the curve $y = \log_e^x$ *at the point* $P(1, 0)$ *is* ________ MHT CET 2019 (02 May Shift 1)

Options:

A. $2x + y = 2$

B. $x - 2y = 1$

C. $x - y = 1$

D. $x + y = 1$

Answer: D

Solution:

Given curve is $y = \log_e x$ and point $p(1, 0)$
then, $\frac{dy}{dx} = \frac{1}{x}$
Hence, equation of normal is $y = -1(x - 1) \Rightarrow x + y = \underline{-1}$

Question285

If $f(x) = 3x^3 - 9x^2 - 27x + 15$, then the maximum value of $f(x)$ is ________ MHT CET 2019 (02 May Shift 1)

Options:

A. -66

B. 30

C. -30

D. 66

Answer: B

Solution:

If $f(x) = 3x^3 - 9x^2 - 27x + 15$
$f'(x) = 9(x^2 - 2x - 3) = 9(x - 3)(x + 1)$
$f(x)$ maximum at $x = -1 \Rightarrow f(-1) = -3 - 9 + 27 + 15 = 30$

Question286

The function $f(x) = x^3 - 3x$ is.... MHT CET 2019 (Shift 2)

Options:

A. Increasing in $(-\infty,-1)\cup(1,\infty)$ and decreasing in $(-1,1)$

B. Increasing in $(0,\infty)$ and decreasing in $(-\infty,0)$

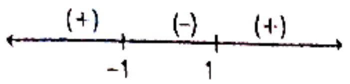
C. Decreasing in $(0,\infty)$ and increasing in $(-\infty,0)$

D. Decreasing in $(-\infty,-1)\cup(1,\infty)$ and increasing in $(-1,1)$

Answer: A

Solution:

We have, function $f(x)=x^3-3x$
$\Rightarrow f'(x)=3x^2-3=3(x+1)(x-1)$



Here, $f'(x)>0 in(-\infty,-1)\cup(1,\infty)$ and $f'(x)<0$ in (-1, 1).
$\therefore f(x)$ is increasing in $(-\infty,-1)$, $\cup(1,\infty)$ and decreasing in (-1, 1).

Question287

Using differentiation, approximate value of $f(x)=x^2-2x+1$ at x = 2.99 is .... MHT CET 2019 (Shift 2)

Options:

A. 3.96

B. 9.96

C. 4.98

D. 5.98

Answer: A

Solution:

Given function, $f(x)=x^2-2x+1$
$\Rightarrow f'(x)=2x-2$
Let x = 3 and $\Delta x=-0.01$
So that $f(x+\Delta x)=f(2.99)$
We know that, $f(x+\Delta x)\approx f(x)+\Delta x f'(x)$
$\approx(x^2-2x+1)+(-0.01)(2x-2)$
Putting x = 3 and $\Delta x=-0.01$, we get
$f(2.99)\approx(3^2-2\times3+1)+(-0.01)(2\times3-2)$
$\approx(4)+(-0.01)(4)$
$\approx 4-0.04\approx 3.96$

Question288

A particle moves so that x = 2 +27t- t^3 . The direction of motion reverses after moving a distance of … units. MHT CET 2019 (Shift 2)

Options:

A. 80

B. 56

C. 60

D. 65

Answer: B

Solution:

We have, $x=2+27t-t^3$
$\Rightarrow \frac{dx}{dt}=27-3t^2$
Since, direction will be reverse,
$\therefore \frac{dx}{dt}=0$
$\Rightarrow 27-3t^2=0$
$\Rightarrow t=3$
$\therefore$ Distance $x=2+27t-t^3$
$=2+27\times 3-(3)^3$
$=2+81-27=56$ units

Question289

The edge of a cube is decreasing at the rate of 0.04 cm/sec. If the edge of the cube is 10 cms, then rate of decrease of surface area of the cube is … MHT CET 2019 (Shift 1)

Options:

A. $\frac{4.8cm^2}{\text{sec}}$

B. $\frac{4.08cm^2}{\text{sec}}$

C. $\frac{48cm^2}{\text{sec}}$

D. $\frac{4.008cm^2}{\text{sec}}$

Answer: A

Solution:

Let edge of a cube be x cm, then surface area of the cube, $A=6x^2$
It is given that, $\frac{dx}{dt}=-0.04$ cm/sec
Now, $\frac{dA}{dt}=12x\frac{dx}{dt}$
$=12x(-0.04)$

$= -0.48x$
when, $x = 10,$ then $\frac{dA}{dt} = 0.48 \times 10 = -4.8 \frac{cm^2}{\sec}$

Question290

The slope of normal to the curve $x = \sqrt{t}$ and $y = t - \frac{1}{\sqrt{t}}$ at $t = 4$ is ... MHT CET 2019 (Shift 1)

Options:

A. $\frac{-17}{4}$

B. $\frac{4}{17}$

C. $\frac{-4}{17}$

D. $\frac{17}{4}$

Answer: C

Solution:

Key Idea Firstly find $\frac{dy}{dx} = \frac{dyldt}{dxldt}$, then
Use slope of normal $= -\frac{1}{\left(\frac{dy}{dx}\right)}$
We have, $x = \sqrt{t}$ and $y = t - \frac{1}{\sqrt{t}}$
Now $\frac{dx}{dt} = \frac{1}{2\sqrt{t}}, \frac{dy}{dt} = 1 + \frac{1}{2}t^{-\frac{3}{2}} \therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1+\frac{1}{2}t^{3/2}}{\frac{1}{2\sqrt{t}}}$
$= 2\sqrt{t}\left(1 + \frac{1}{2}t^{-\frac{3}{2}}\right)$
$= 2\sqrt{t}\left(1 + \frac{1}{2t^{3/2}}\right)$
$= \frac{2t^{3/2}+1}{t}$
At $t = 4, \frac{dy}{dx} = \frac{2(4)^{3/2}+1}{4} = \frac{17}{4}$
$\therefore$ Slope of normal at $t = 4, -\frac{1}{\left(\frac{dy}{dx}\right)} = -\frac{4}{17}$

Question291

If $f(x) = x + \frac{1}{x}, x \neq 0$, then local maximum and minimum values of function f are respectively.... MHT CET 2019 (Shift 1)

Options:

A. – 1 and 1

B. – 2 and 2

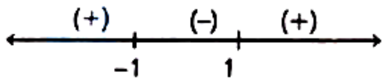
C. 2 and – 2

D. 1 and – 1

Answer: B

Solution:

We have, $f(x) = x + \frac{1}{x}, x \neq 0$
On differentiating w.r.t.x, we get
$f'\left(x\right) = 1 - \frac{1}{x^2} f''\left(x\right) =$
For maxima or minima, we put $f'(x) = 0$
$\Rightarrow 1 - \frac{1}{x^2} = 0$
$\Rightarrow x^2 - 1 = 0 \Rightarrow x = -1, 1$
Now, since $f'(x) > 0, x \in (-1 - h, -1)$ and $f'(x) < 0, x \in (-1, -1 + h)$



$\Rightarrow x = -1$ is point of local maxima.
and local maxima value is $f(-1) = -1 + \frac{1}{(-1)} = -2$
and $f'(x) < 0$ when $x \in (1 - h, 1)$
and $f''(x) > 0$ when $x \in (1, 1 + h)$
$\Rightarrow x = 1$ Is point of minima.
and local minima value, $f(1) = 1 + \frac{1}{1} = 2$

Question292

If $f(x) = \frac{x}{x^2+1}$ is increasing function then the value of x lies in MHT CET 2018

Options:

A. R

B. $(-\infty, \ -1)$

C. $(1, \ \infty)$

D. $(-1, \ 1)$

Answer: D

Solution:

$f(x) = \frac{x}{x^2+1}$
$f'(x) = \frac{(x^2+1) - x(2x)}{(x^2+1)^2} = \frac{-x^2+1}{(x^2+1)^2}$
Since f(x) is increasing function,
$f'(x) > 0$
$\therefore \left(-x^2 + 1\right) > 0$ as $x^2 + 1$ is always positive
$\therefore x^2 - 1 < 0$
$\therefore (x - 1)(x + 1) < 0$
$\therefore x \in (-1, 1)$

Question293

If the line $y=4x-5$ touches to the curve $y^2=ax^3+b$ at the point $(2,\ 3)$ then $7a+2b=$ MHT CET 2018

Options:

A. 0

B. 1

C. -1

D. 2

Answer: A

Solution:

$$y^2=ax^3+b$$
$$\Rightarrow 2y\frac{dy}{dx}=a3\left(x^2\right)$$
$$\Rightarrow \left.\frac{dy}{dx}\right|_{(2,3)}=\frac{a(3)(2)^2}{2(3)}=2a=\text{Slope of tangent}$$
Since given line $y=4x-5$ has slope
$$m=4$$
$$\Rightarrow 2a=4\Rightarrow a=2$$
Now since $(2,\ 3)$ lies on curve $y^2=ax^3+b$
$$\Rightarrow (3)^2=a(2)^3+b$$
$$\Rightarrow 9=8a+b$$
$$\Rightarrow 9-16=b$$
$$\Rightarrow b=-7$$
So $7a+2b$
$$=7\,(2)+2\,(-7)$$
$$=14-14$$
$$=0$$

Question294

The minimum value of the function $f(x)=x\log x$ is MHT CET 2018

Options:

A. $-\frac{1}{e}$

B. $-e$

C. $\frac{1}{e}$

D. e

Answer: A

Solution:

$f(x) = x \log x$
$\Rightarrow f'(x) = x \times \frac{1}{x} + \log x \times 1$
$\Rightarrow f'(x) = 1 + \log x$
Now for f(x) to be minimum,
$f'(x) = 0$
$\Rightarrow 1 + \log x = 0$
$\Rightarrow \log_e x = -1$
$\Rightarrow x = e^{-1} = \frac{1}{e}$
Also $f''(x) = \frac{1}{x}$
$\Rightarrow f''\left(\frac{1}{e}\right) = \frac{1}{\frac{1}{e}} = 3 > 0$
$\Rightarrow f(x)$ is minimum at $x = \frac{1}{\mathrm{e}}$ and the minimum value is $f\left(\frac{1}{e}\right) = \frac{1}{e}\log\frac{1}{e} = \frac{-1}{e}$

Question295

The maximum value of $f(x)= \frac{\log x}{x}\ \left(x \neq 0, x \neq 1\right)$ is MHT CET 2017

Options:

A. e

B. $\frac{1}{e}$

C. e^2

D. $\frac{1}{e^2}$

Answer: B

Solution:

$f(x)= \frac{\log x}{x}$
$f'(x) = \frac{x\frac{1}{x} - \log x}{x^2} = \frac{1 - \log x}{x^2}$
$f'(x) = 0 \Rightarrow \frac{1-\log x}{x^2} = 0$
$\Rightarrow \log x = 1$
$\Rightarrow x = e$
$\Rightarrow$ Max value is $f(e)= \frac{\log e}{e} = \frac{1}{e}$

Question296

The point on the curve $y = \sqrt{x-1}$ where the tangent is perpendicular to the line $2x + y - 5 = 0$ is MHT CET 2017

Options:

A. $(2, -1)$

B. $(10, 3)$

C. $(2, 1)$

D. $(5,-2)$

Answer: C

Solution:

For given curve,
$\frac{dy}{dx}=\frac{1}{2\sqrt{x-1}}=m_1(\text{say})$
Now Slope of line $2x+y-5=0$ is $m_2=-2$
Since lines are perpendicular
$\Rightarrow m_1m_2=-1$
$\Rightarrow\left(\frac{1}{2\sqrt{x-1}}\right)(-2)=-1$
$\Rightarrow\frac{2}{2\sqrt{x-1}}=1$
$\Rightarrow\sqrt{x-1}=1$
$\Rightarrow$ Squaring both sides, $x-1=1$
$\Rightarrow x=2$
$\Rightarrow\quad y=\sqrt{x-1}$
$=\sqrt{2-1}$
$=\sqrt{1}=1$
So the required point is $\quad(2,\ 1)$

Question297

If the volume of spherical ball is increasing at the rate of $4\pi\ cm^3/\sec$ then the rate of change of its surface area when the volume is $288\pi\,cm^3$ is MHT CET 2017

Options:

A. $\frac{4}{3}\pi\ cm^2/sec$

B. $\frac{2}{3}\pi\ cm^2/sec$

C. $4\ \pi\ cm^2/sec$

D. $2\pi\ cm^2/sec$

Answer: A

Solution:

$V=\frac{4}{3}\pi r^3\Rightarrow\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$
When $V-288\pi$
$288\ \pi=\frac{4}{3}\pi r^3\Rightarrow r=6$
Now $\frac{dV}{dt}=4\pi$
$\therefore\quad 4\pi r^2\frac{dr}{dt}=4\pi=\frac{dr}{dt}=\frac{1}{r^2}$

A = surface area $= 4\pi r^2$

$\therefore \frac{dA}{dt} = 8\pi r \frac{dr}{dt} = 8\pi r \times \frac{1}{r^2} = \frac{8\pi}{r} = \frac{8\pi}{6} = \frac{4\pi}{3}$

Question298

The objective function of LPP defined over the convex set attains its optimum value at MHT CET 2017

Options:

A. At least two of the corner points

B. All the corner points

C. At least one of the corner points

D. None of the corner points

Answer: C

Solution:

Let $Z = ax +$ by be the objective function

When Z has optimum value(maximum or minimum), where the variables x and y are subject to constraints described by linear inequalities, this optimum value must occur at a corner points of the feasible region.

Thus, the function attains its optimum value at one of the corner points.

Question299

If Rolle's theorem for a function $f(x) = e^x(\sin x - \cos x)$ is verified on $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$ then the value of c is MHT CET 2016

Options:

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{3\pi}{4}$

D. π

Answer: D

Solution:

Given, $f(x) = e^x(\sin\ x - \cos\ x)$

$\therefore\ \ f'(x) = e^x[\cos\ x + \sin\ x] + [\sin\ x - \cos\ x]e^x$

$\therefore\ \ f'(x) = 2e^x \sin\ x$

To verify Rolle's Theorem.

$f'(c) = 0$

$2e^c \sin\ c = 0$

$\Rightarrow \sin c = 0$
$\therefore c = \pi$

Question300

The approximate value of $f(x) = x^3 + 5x^2 - 7x + 9$ at $x = 1.1$ is MHT CET 2016

Options:

A. 8.6

B. 8.5

C. 8.4

D. 8.3

Answer: A

Solution:

$f(x+h) = f(x) + h\, f'(x)$
Here, $x = 1,\ h = 0.1,\ f'(x) = 3x^2 + 10x - 7$
$\therefore f(1+0.1) = f(1) + 0.1 \times f'(1)$
$= 8 + 0.1 \times 6$
$= 8.6$

Question301

The point on the curve $6y = x^3 + 2$ at which $y-$ co-ordinate is changing 8 times as fast as $x-$ co-ordinate is ________ MHT CET 2016

Options:

A. $(4, 11)$

B. $(4, -11)$

C. $(-4, 11)$

D. $(-4, -11)$

Answer: A

Solution:

$\frac{dy}{dx} = \frac{x^2}{2} = 8$
$\Rightarrow x = \pm 4$
Substituting value of x in the curve, we get $(4,\ 11),\ \left(-4,\ -\frac{31}{3}\right)$

Question302

Divide 10 into two parts such that the sum of double of the first and the square of the second is minimum MHT CET 2012

Options:

A. $(6,4)$

B. $(7,3)$

C. $(8,2)$

D. $(9,1)$

Answer: D

Solution:

Let x and y be the two parts of the number 10 .

$\therefore$

$x+y=10 \ldots$ (i)

According to the question, Let
$$\begin{aligned} A &= 2x+y^2 \\ &= 2x+(10-x)^2 \\ &= 2x+100+x^2-20x \\ &= x^2-18x+100 \end{aligned}$$

On differentiating w.r.t. x, we get $\frac{dA}{dx}=2x-18$

For max or min of A, Put $\frac{dA}{dx}=0=2x-18$

$\Rightarrow$

$x=9$

Now, $\frac{d^2A}{dx^2}=2>0 \quad (\min)$

On putting $x=9$ in Eq. (i), we get $y=1$

$\therefore \quad (x,y)=(9,1)$

Question303

All the points on the curve $y^2=4a|x+a\sin(x/a)|$, where the tangent is parallel to the axis of x are lies on MHT CET 2012

Options:

A. circle

B. parabola

C. straight line

D. None of these

Answer: B

Solution:

$y^2 = 4a\left[x + a\sin\left(\frac{x}{a}\right)\right]$ ...(i)

$\therefore \quad 2y\frac{dy}{dx} = 4a\left[1 + \cos\left(\frac{x}{a}\right)\right]$ ...(ii)

If tangent is parallel to x -axis, then $\frac{dy}{dx} = 0$

So, from Eq. (i), we get $\cos\left(\frac{x}{a}\right) = -1$

$\therefore$

$$\sin\left(\frac{x}{a}\right) = 0$$

On putting this value in Eq. (i), we get $y^2 = 4a(x+0) \Rightarrow y^2 = 4ax$

So, all the points on the curve

$$y^2 = 4a\left(x + a\sin\frac{x}{a}\right)$$

where the tangent is parallel to the x -axis are lies on parabola.

Question304

The length of normal at any point to the curve, $y = c\cosh\left(\frac{x}{c}\right)$ is MHT CET 2012

Options:

A. fixed

B. $\frac{y^2}{c^2}$

C. $\frac{y^2}{c}$

D. $\frac{y}{c^2}$

Answer: C

Solution:

Given, $y=c\cosh\left(\frac{x}{c}\right)$...(i)

$$\frac{dy}{dx}=c\cdot\frac{1}{c}\cdot\sinh\left(\frac{x}{c}\right)=\sinh\left(\frac{x}{c}\right)$$

Now, length of normal

$$=y\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$=c\cosh\left(\frac{x}{c}\right)\sqrt{1+\sin^2\ h\left(\frac{x}{c}\right)}$

$=c\cosh\left(\frac{x}{c}\right)\sqrt{\cosh^2\left(\frac{x}{c}\right)}$

$=c\left[\cosh\left(\frac{x}{c}\right)\right]^2$

$=c\left(\frac{y}{c}\right)^2$ [from Eq. (i)]

$=\frac{y^2}{c}$

Question305

The height of right circular cylinder of maximum volume inscribed in a sphere of diameter $2a$ is MHT CET 2012

Options:

A. $2\sqrt{3}a$

B. $\sqrt{3}a$

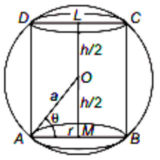
C. $\frac{2a}{\sqrt{3}}$

D. $\frac{a}{\sqrt{3}}$

Answer: C

Solution:

Let the radius and height of the cylinder are and h, respectively.



In $\triangle AOM$

$\therefore$

$$r^2+\left(\frac{h^2}{4}\right)=a^2$$
$$h^2=4\left(a^2-r^2\right)$$

Now, $V=\pi r^2 h=\pi\left(a^2 h-\frac{1}{4} h^3\right)$

For max or min,

$$\frac{dV}{dh}=\pi\left(a^2-\frac{3}{4} h^2\right)=0$$
$$\Rightarrow h=\left(\frac{2}{\sqrt{3}}\right) a$$

Now, $\frac{d^2 V}{dh^2}=-\frac{6h}{4}<0$

So, V is maximum at $h=\frac{2a}{\sqrt{3}}$.

Question306

For all real x, the minimum value of $\frac{1-x+x^2}{1+x+x^2}$ is MHT CET 2011

Options:

A. 0

B. 1/3

C. 1

D. 3

Answer: B

Solution:

Let $y=\frac{1-x+x^2}{1+x+x^2}=1-\frac{2x}{1+x+x^2}$

$$=1-\frac{2}{\frac{1}{x}+1+x}$$
$$\Rightarrow \quad y=1-\frac{2}{t}$$

where

$$t=\frac{1}{x}+1+x$$

Now, y is minimum, when $\frac{2}{t}$ is max $\Rightarrow t$ is min.

$$\therefore \quad \frac{dt}{dx} = -\frac{1}{x^2} + 1 = 0$$

$$\Rightarrow \quad x = \pm 1$$

$$\frac{d^2t}{dx^2} = \frac{2}{x^3} > 0, \text{ for } x = 1$$

$\therefore$ Minimum value of y is

$$1 - \frac{2}{1+1+1} = 1 - \frac{2}{3}$$
$$= \frac{1}{3}$$

Question307

If $x + y = k$ is normal to $y^2 = 12x$, then k is MHT CET 2011

Options:

A. 3

B. 9

C. -9

D. -3

Answer: B

Solution:

Let $x+y=k$ be normal to

$$y^2=12x \text{ at } P(\alpha,\beta)$$

$\therefore \quad \beta^2=12\alpha$

Also, slope of normal at $P(\alpha,\beta)$ is -1

Form, Eq.(i),

$$\frac{dy}{dx}=\frac{6}{y}$$
$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_P=\frac{6}{\beta}$$

$\therefore$

$$-1=\frac{-1}{6/\beta}$$

$\Rightarrow$

$\beta=6, \alpha=3$

$\therefore P$ is $(3,6)$ which lies on $x+y=k$

$\therefore \quad 3+6=k$

$\Rightarrow \quad k=9$

Question308

The equation of tangent to the curve given by $x=3\cos\theta, y=3\sin\theta$ at $\theta=\frac{\pi}{4}$ is MHT CET 2010

Options:

A. $x+y=\sqrt{2}$

B. $3x+y=3\sqrt{2}$

C. $x+y=3\sqrt{2}$

D. $x+3y=3\sqrt{2}$

Answer: C

Solution:

Given, $x=3\cos\theta, y=3\sin\theta$. On squaring and

adding we get $x^2+y^2=9$, which represent a circle.

Equation of tangent at $\theta=\frac{\pi}{4}$ is

$$\begin{aligned} & x\cdot\left(3\cos\frac{\pi}{4}\right)+y\cdot\left(3\sin\frac{\pi}{4}\right)=9 \\ \Rightarrow\quad & x+y=3\sqrt{2} \end{aligned}$$

Question309

The equation of tangent to the curve $y^2=ax^2+b$ at point $(2,3)$ is $y=4x-5$, then the values of a and $\bar{b}$ are MHT CET 2010

Options:

A. $3,-5$

B. $6,-5$

C. 6,15

D. $6,-15$

Answer: D

Solution:

Given, $\quad y^2=ax^2+b^2$

$\therefore$

$2y\frac{dy}{dx}=2ax$

$\Rightarrow\quad \frac{dy}{dx}=\frac{ax}{y}$

$\therefore$ Slope at $(2,3)=\left(\frac{dy}{dx}\right)_{(2,3)}=\frac{2a}{3}$

But slope of given tangent is $m=4\ \frac{2a}{3}=4\Rightarrow a=6$

Since, point $(2,3)$ lies on the curve so, it satisfies the equation of the curve

$\therefore$

$(3)^2=6(2)^2+b$

$\Rightarrow\quad b=-15$

Question310

The approximate value of $\sqrt[3]{28}$ is MHT CET 2009

Options:

A. 3.0037

B. 3.037

C. 3.0086

D. 3.37

Answer: B

Solution:

Let $y = \sqrt[3]{28}$

Taking log on both sides, we get

$= 3.037$ (approximately)

Question311

The equation of the tangent to the curve $y = 4xe^x$ at $\left(-1, \frac{-4}{e}\right)$ is MHT CET 2009

Options:

A. $y = -1$

B. $y = -\frac{4}{e}$

C. $x = -1$

D. $x = \frac{-4}{e}$

Answer: B

Solution:

iven curve is $y = 4xe^x$ $\frac{dy}{dx} = 4e^x + 4xe^x$

At $\left(-1, -\frac{4}{e}\right)$ $\left(\frac{dy}{dx}\right)_{(-1,-4/e)} = 4e^{-1} + 4(-1)e^{-1} = 0$

$\therefore$ Equation of tangent is $\left(y + \frac{4}{e}\right) = 0(x+1)$

$\Rightarrow y = -\frac{4}{e}$

Question312

The maximum value of function $x^3 - 12x^2 + 36x + 17$ in the interval $[1, 10]$ is MHT CET 2008

Options:

A. 17

B. 177

C. 77

D. None of these

Answer: B

Solution:

Let $f(x) = x^3 - 12x^2 + 36x + 17$

$\therefore f'(x) = 3x^2 - 24x + 36 = 0$

For maxima, put $f'(x) = 0 \Rightarrow \quad 3x^2 - 24x + 36 = 0$

$\Rightarrow \quad (x-2)(x-6) = 0$

$\Rightarrow \quad x = 2, 6$

Again, $f''(x) = 6x - 24$ is negative at $x = 2$ So that, $f(6) = 17, f(2) = 49$

At the end points, $f(1) = 42, f(10) = 177$ So that, $f(x)$ has its maximum value 177 .

Question313

Angle of intersection of the curve $r = \sin\theta + \cos\theta$ and $r = 2\sin\theta$ is equal to MHT CET 2008

Options:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{4}$

D. None of these

Answer: C

Solution:

Given, $r = \sin\theta + \cos\theta$ and $r = 2\sin\theta$

$$\begin{aligned} \therefore \quad & 2\sin\theta = \sin\theta + \cos\theta \\ \Rightarrow \quad & \sin\theta = \cos\theta \\ \Rightarrow \quad & \tan\theta = 1 \\ \Rightarrow \quad & \theta = \frac{\pi}{4} \end{aligned}$$

Question314

The equation of the tangent parallel to $y - x + 5 = 0$ drawn to $\frac{x^2}{3} - \frac{y^2}{2} = 1$ is MHT CET 2008

Options:

A. $x - y - 1 = 0$

B. $x - y + 2 = 0$

C. $x + y - 1 = 0$

D. $x + y + 2 = 0$

Answer: A

Solution:

Given hyperbola is $\frac{x^2}{3}-\frac{y^2}{2}=1$ ...(i)Equation of tangent parallel to $y-x+5=0$ is

$$y-x+\lambda=0$$
$$\Rightarrow \quad y=x-\lambda \ldots(ii)$$

If line (ii) is a tangent to hyperbola (i), then

$$-\lambda= \pm \sqrt{3x-2}$$
$$\left(\text{ from } c= \pm \sqrt{a^2 m^2-b^2}\right)$$
$$\Rightarrow \quad -\lambda= \pm 1$$
$$\Rightarrow \quad \lambda=-1,+1$$

Put the values of λ in Eq. (ii), we get $x-y-1=0$ and $x-y+1=0$ are the required tangents.

Question315

The point of the curve $y^2=2(x-3)$ at which the normal is parallel to the line $y-2x+1=0$ is MHT CET 2008

Options:

A. $(5,2)$

B. $\left(-\frac{1}{2},-2\right)$

C. $(5,-2)$

D. $\left(\frac{3}{2}, 2\right)$

Answer: C

Solution:

Given, $y^2 = 2(x-3)$ ...(i)

On differentiating w.r.t. x, we get $2y\frac{dy}{dx} = 2$

$\Rightarrow$

$\frac{dy}{dx} = \frac{1}{y}$

Slope of the normal $= \frac{-1}{(dy/dx)} = -y$ Slope of the given line $= 2 \therefore \quad y = -2$

From Eq. (i), $x = 5$

$\therefore$ Required point is $(5, -2)$.

Question316

The abscissae of the points, where the tangent to curve $y = x^3 - 3x^2 - 9x + 5$ is parallel to x -axis, are MHT CET 2008

Options:

A. $x = 0$ and 0

B. $x = 1$ and -1

C. $x = 1$ and -3

D. $x = -1$ and 3

Answer: D

Solution:

Given, $y = x^3 - 3x^2 - 9x + 5$

$\Rightarrow \quad \frac{dy}{dx} = 3x^2 - 6x - 9$

We know that, this equation gives the slope of the tangent to the curve. The tangent is parallel to x -axis,
$\frac{dy}{dx} = 0$

$\Rightarrow \quad 3x^2 - 6x - 9 = 0$

$\Rightarrow \quad x = -1, 3$

Question317

The slope of the tangent at (x, y) to a curve passing through $\left(1, \frac{\pi}{4}\right)$ is given by $\frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$, then the equation of the curve is MHT CET 2007

Options:

A. $y = \tan^{-1}\left[\log\left(\frac{e}{x}\right)\right]$

B. $y = x\tan^{-1}\left[\log\left(\frac{x}{e}\right)\right]$

C. $y = x\tan^{-1}\left[\log\left(\frac{e}{x}\right)\right]$

D. None of the above

Answer: C

Solution:

According to the given condition,

$$\frac{dy}{dx}=\frac{y}{x}-\cos^2\left(\frac{y}{x}\right)$$

On putting $y=vx$

$$\Rightarrow \quad \frac{dy}{dx}=v+x\frac{dv}{dx}, \text{ we get}$$

$$v+x\frac{dv}{dx}=v-\cos^2 v$$

$$\Rightarrow\frac{dv}{\cos^2 v}=-\frac{dx}{x}$$

$$\Rightarrow\sec^2 v dv=\frac{-1}{x}dx$$

On integrating both sides, we get

$$\tan v=-\log x+\log c$$

$$\Rightarrow \tan\left(\frac{y}{x}\right)=-\log x+\log c$$

Since, this curve is passing through $(1,\pi/4)$.

$$\therefore \quad \tan\left(\frac{\pi}{4}\right)=-\log 1+\log c\Rightarrow \log c=1$$

$$\therefore \quad \tan\left(\frac{y}{x}\right)=-\log x+1$$

$$\Rightarrow \tan\left(\frac{y}{x}\right)=-\log x+\log e$$

$$\Rightarrow \qquad y=x\tan^{-1}\left[\log\left(\frac{e}{x}\right)\right]$$

Question318

If the function $f(x)=2x^3-9ax^2+12a^2x+1$ attains its maximum and minimum at p and q respectively such that $p^2=q$, then a equals MHT CET 2007

Options:

A. 0

B. 1

C. 2

D. None of these

Answer: C

Solution:

Given, $f(x)=2x^3-9ax^2+12a^2x+1$ attains maximum and minimum at p and q respectively. $\therefore$

$$f'(p)=0, f'(q)=0$$

Now,

$$f''(p)<0 \text{ and } f''(q)>0$$
$$f'(p)=0$$

and

$$f'(q)=0$$
$$6p^2-18ap+12a^2=0$$
$$6q^2-18aq+12a^2=0$$

and

$$\Rightarrow \quad p^2-3ap+2a^2=0$$
$$\text{and} \quad q^2-3aq+2a^2=0$$

$p=a, 2a, q=a, 2a$

$$\Rightarrow \quad p=a, 2a, q=a,$$
$$\text{Now,} \quad f''(p)<0$$

$\Rightarrow \quad 12p-18a<0$

$\Rightarrow \quad p<\frac{3}{2}a$

and $f''(q)>0 \Rightarrow 12q-18a>0$

$\Rightarrow \quad q>\frac{3}{2}a$

From Eqs. (i), (ii) and (iii), we get $p=a, q=2a$

$$\text{Now,} \quad p^2=q$$

$$\Rightarrow \quad a^2=2a$$
$$\Rightarrow \quad a=0,2$$

But for $a=0, f(x)=2x^3+1$ which does not

attains a maximum or minimum for any value of x. Hence, $a=2$.

Question319

On the interval $[0,1]$ the function $x^{25}(1-x)^{75}$ takes its maximum value at the point MHT CET 2007

Options:

A. 0

B. $1/4$

C. $1/2$

D. $1/3$

Answer: B

Solution:

Given, $f(x)=x^{25}(1-x)^{75}$

$$\begin{aligned} \Rightarrow f'(x) &= 25x^{24}(1-x)^{75}-75x^{25}(1-x)^{74} \\ &= 25x^{24}(1-x)^{74}(1-4x) \\ \therefore \quad f'(x) &= 0 \\ \Rightarrow \quad x &= 0,1,1/4 \end{aligned}$$

If $x<1/4$, then

$$f'(x)=25x^{24}(1-x)^{74}(1-4x)>0.$$

and if $x>1/4$, then

$$f'(x)=25x^{24}(1-x)^{74}(1-4x)<0.$$

Thus, $f'(x)$ changes its sign from positive to negative as x passes through $1/4$ from left to right. Hence, $f(x)$ attains its maximum at $x=1/4$.

Question320

The function $f(x)=\log(1+x)-\frac{2x}{2+x}$ is increasing on MHT CET 2007

Options:

A. $(0,\infty)$

B. $(-\infty,0)$

C. $(-\infty,\infty)$

D. None of these

Answer: A

Solution:

Given, $f(x) = \log(1+x) - \frac{2x}{2+x}$

$$\therefore \quad f'(x) = \frac{1}{1+x} - \frac{(2+x)\cdot 2 - 2x}{(2+x)^2}$$
$$= \frac{x^2}{(1+x)(x+2)^2}$$

Clearly, $f'(x) > 0$ for all $x > 0$. Hence, $f(x)$ is increasing on $(0, \infty)$.

Question321

If $f(x) = kx - \sin x$ is monotonically increasing, then MHT CET 2007

Options:

A. $k > 1$

B. $k > -1$

C. $k < 1$

D. $k < -1$

Answer: A

Solution:

Since, $f(x) = kx - \sin x$ is monotonically increasing for all $x \in R$. Therefore,

$$f'(x) > 0 \text{ for all } x \in R$$

$\therefore \quad f'(0) > 0$

$$\Rightarrow \quad k - \cos 0 > 0$$
$$\Rightarrow \quad k > 1$$

Question322

The positive root of $x^2 - 78.8 = 0$ after first approximation by Newton Raphson method assuming initial approximation to the root is 14, is MHT CET 2007

Options:

A. 9.821

B. 9.814

C. 9.715

D. 9.915

Answer: B

Solution:

Here, $\quad x_0 = 14, f(x) = x^2 - 78.8 \text{and } f'(x) = 2x$

$$\therefore \quad x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$
$$= 14 - \frac{(14)^2 - (78.8)}{2 \times 14} = 9.814$$

Question323

The maximum area of the rectangle that can be inscribed in a circle of radius r, is MHT CET 2007

Options:

A. πr^2

B. r^2

C. $\pi r^2/4$

D. $2r^2$

Answer: D

Solution:

Area of rectangle,

$$\begin{aligned} A &= 2x \cdot 2\sqrt{r^2 - x^2} \\ &= 4x\sqrt{r^2 - x^2} \\ \frac{dA}{dx} &= \frac{4\left(r^2 - 2x^2\right)}{\sqrt{r^2 - x^2}} \end{aligned}$$

For maximum or minimum put $\frac{dA}{dx} = 0$

$$\Rightarrow \quad x = r/\sqrt{2}$$

It can be easily checked that $\frac{d^2A}{dx^2} < 0$ for this value of x. $\therefore A$ is maximum for $x = \frac{r}{\sqrt{2}}$ and the maximum value of A is given by

$$A = 4\frac{r}{\sqrt{2}}\sqrt{r^2 - \frac{r^2}{2}} = 2r^2$$
